

Analysis of a Microstrip Leaky-Wave Antenna Loaded with Shorted Stubs

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Abstract—A new theory is provided to analyze a novel microstrip leaky-wave antenna (MLWA) with shorted stub loading, used for fixed-frequency beam steering applications. Formulas for calculating the radiation angle of MLWA with stub loading are deduced. The radiation angle estimated theoretically agrees very well with the rigorous numerical result calculated with the finite difference time domain (FDTD) method. This new theory can be used to explain the MLWA behavior with any reactive element loading, if the input admittance of the reactive element is known.

Index Terms—Microstrip leaky-wave antenna, radiation angle, stub.

I. INTRODUCTION

THE microstrip leaky-wave antenna (MLWA) has a low profile, minimal weight, simple fabrication, and the extraordinary advantages of narrow beamwidth and frequency scanning characteristics [1], [2]. The MLWA [shown in Fig. 1(a)] operates in the first higher order mode, TE_{01} . The radiation characteristics of the MLWA can be explained by its propagation constant $k_z = \beta_z - j\alpha_z$, where β_z is the phase constant and α_z is the attenuation constant. The radiation angle θ_r can be estimated [1], [2]

$$\theta_r = \sin^{-1}(\beta_z/k_0) \quad (1)$$

where k_0 is the wave number in free space, θ_r is the main beam direction angle measured from the normal direction (y axis). Several rigorous methods have been used to obtain the k_z , such as the transverse-resonance method in conjunction with a Wiener-Hopf approach [1], the method of moments [3], the finite difference time domain (FDTD) method [4] but these methods are complicated and time-consuming. A simple method proposed by Bhattacharyya [5] can estimate the complex propagation constant quickly by solving the wave equation with impedance boundary conditions but it has to solve a transcendental equation with a numerical iteration technique. The formulas deduced in [4] can be used to calculate the k_z value conveniently and the results are more accurate than those calculated with the Bhattacharyya method [5].

Unfortunately, the frequency dependence of the scanning angle of MLWA limits the use of MLWA, when fixed frequency or narrow frequency bandwidth and wide angle coverage is

demanded. Fixed-frequency beam-steering MLWA is the right substitute. Since 2000, a lot of research has been dedicated to the fixed-frequency beam-steering MLWA [6]–[13]. The MLWA loaded with capacitors, proposed by Luxey [6], is a good method for the construction of a fixed-frequency beam steering MLWA. Luxey mentioned that the radiation angle of MLWA could be changed by placing reactive elements along the radiating edges of the MLWA in a letter [6], where capacitors and inductors are used as reactive elements. He pointed out that the equivalent extension of the MLWA width ΔL would increase when the MLWA was loaded with capacitors, and the ΔL would decrease when the MLWA was loaded with inductors. He also showed the variation of β_z as a function of ΔL . But the variation of ΔL was not calculated [6] when the MLWA was loaded with reactive elements, so the radiation angle could not be estimated accurately.

In this letter, a new theory is provided for analyzing a novel MLWA which is loaded with shorted stubs, as shown in Fig. 1(b) and Fig. 1(c). This novel MLWA might be used for fixed-frequency beam steering application of MLWA. With the shorted stubs loading the edges of MLWA, the β_z of MLWA will be modified, resulting in the variation of the radiation angle θ_r . The variation of θ_r can be calculated not only theoretically, but also numerically with the FDTD method. The theoretical results agree very well with the numerical results. But the theoretical calculation is much faster than the numerical simulation. Besides, this theory can explain the MLWA loaded with any reactive element, only if the input admittance of the reactive element is known. So, this theory is very useful for the design of the MLWA with reactive element loading.

II. MLWA WITH SHORTED STUB LOADING

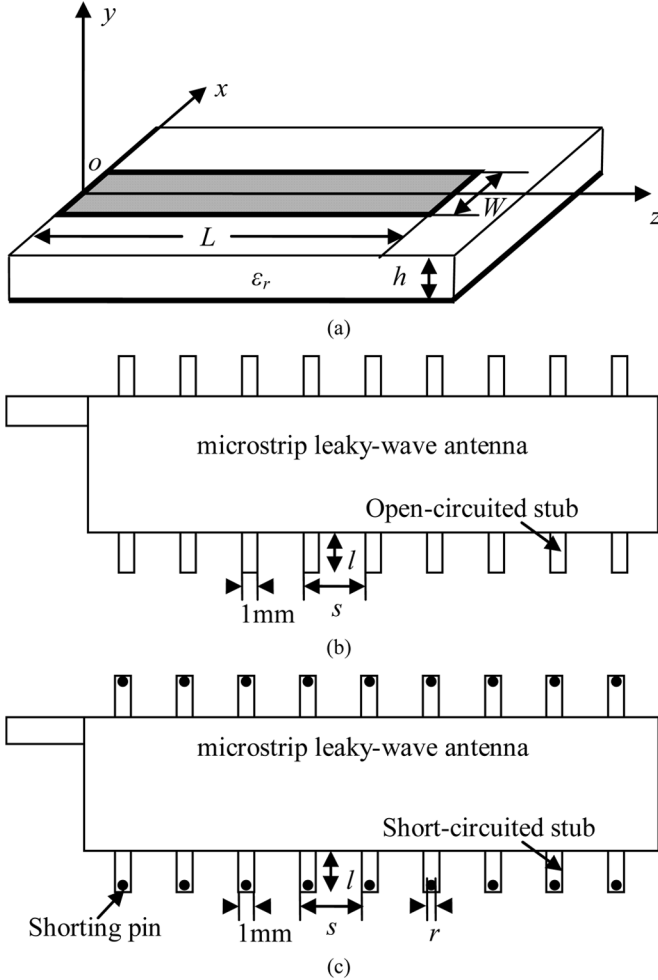
The structure of the MLWA is shown in Fig. 1(a), where L and W are the length and width of MLWA, h and ϵ_r are the thickness and relative permittivity of the substrate, respectively. To verify the accuracy of the FDTD codes used in this letter, the MLWA in [14] is first simulated. The far-field pattern is shown in Fig. 2 and compared with the measured data from [14]. It should be noted that the radiation angle θ in [14] is the main beam direction angle measured from z axis, and the radiation angle θ_r here is measured from y axis, so $\theta_r = 90^\circ - \theta$. It is found that the simulation results agree very well with the measured data. So we believe that the FDTD method can be used to predict the far-field pattern of MLWA precisely.

The MLWA with open-circuited stub loading is shown in Fig. 1(b), where l is the length of the short stub, s is the distance between two adjacent stubs. The open-circuited stub is a short lossless microstrip line with a length $l < \lambda/4$, where λ is the wave length in the shorted stub. The open-circuited stub

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$L=120\text{mm}$, $W=11\text{mm}$, $h=0.508\text{mm}$, $\epsilon_r=2.2$

Fig. 1. The structure of the MLWA. (a) The MLWA without stub loading. (b) The MLWA with open-circuited stub loading. (c) The MLWA with short-circuited stub loading.

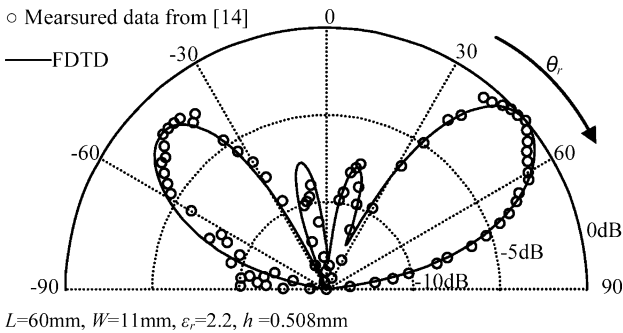


Fig. 2. Radiation patterns of MLWA in [14] at 10.5 GHz.

can be used as a shunt capacitor [15]. The far-field patterns of the MLWA with open-circuited stub loading are calculated with the FDTD method and shown in Fig. 3. It shows that the radiation angle θ_r of MLWA loaded with open-circuited stubs can be increased to 46° compared to 15° of the MLWA without stub loading. θ_r decreases along with the increase of s .

The MLWA with short-circuited stub loading is shown in Fig. 1(c). The short-circuited stub is shorted by a pin with a

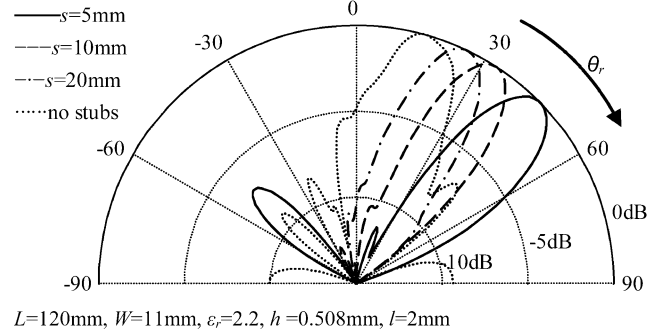


Fig. 3. Radiation patterns of MLWA with open-circuited stub loading at 8.5 GHz.

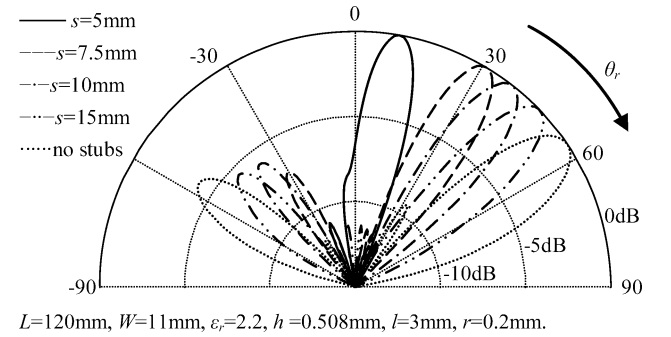


Fig. 4. Radiation patterns of MLWA with short-circuited stub loading at frequency 10.5 GHz.

radius r . The short-circuited stub can be used as a shunt inductor [15]. The far-field pattern of MLWA with short-circuited stub loading is calculated with the FDTD method and shown in Fig. 4. It is found that the radiation angle θ_r of the MLWA with short-circuited stub loading can be decreased to 10° comparing to 56° of the MLWA without stub loading. θ_r increases along with the increase of s .

This kind of MLWA with shorted stub loading provides a new idea for constructing a fixed-frequency beam-steering MLWA, if the distance s between two adjacent connecting stubs can be managed. If there are electrical elements to control the connection between the MLWA and the shorted stubs (if the MLWA and the shorted stubs are not connected directly, but connected by electrical elements), by switching the connection between the MLWA and the shorted stubs, the distance s between two adjacent connecting stubs can be changed, which would result in the change of the radiation angle of MLWA.

III. THEORETICAL ANALYSIS

In [4], a complex effective width W_e is introduced to analyze the microstrip line in its first higher-order mode:

$$\begin{aligned}
 W_e &= W + 2(\Delta W - jb) = W + 2 \left(-j \frac{120\pi h}{k_0 \epsilon_{re}} \right) \times Y_w \\
 &= W + 2 \left(-j \frac{120\pi h}{k_0 \epsilon_{re}} \right) \times \left(\frac{1}{120\lambda_0} + j \frac{k_0 \epsilon_{re} \Delta W}{120\pi h} \right) \\
 &= W + 2\Delta W - j \frac{h}{\epsilon_{re}}
 \end{aligned} \tag{2}$$

where ΔW is the equivalent strip width extension, b is a parameter introduced to describe the leakage of the microstrip line, Y_w is the radiation admittance per unit length of the microstrip line, ϵ_{re} is the effective relative permittivity, all of ΔW , b , Y_w and ϵ_{re} are referred to [4].

According to waveguide theory, the complex propagation constant $k_z = \beta_z - j\alpha_z$ can be written as [4]

$$k_z = \sqrt{k_0^2 \epsilon_{re} - (\pi/W_e)^2}. \quad (3)$$

When the microstrip line is loaded periodically with reactive elements, (2) should be changed to

$$\begin{aligned} W_e &= W + 2 \left(-j \frac{120\pi h}{k_0 \epsilon_{re}} \right) \left(Y_w + \frac{1}{s} Y_{in} \right) \\ &= W + 2\Delta W - j \frac{h}{\epsilon_{re}} - j \frac{240\pi h}{k_0 \epsilon_{re} s} Y_{in} \end{aligned} \quad (4)$$

where Y_{in} is the input admittance of the reactive element, s is the distance between two adjacent elements. Equation (4) is modified because the input admittance of the reactive element per unit length Y_{in}/s can be treated as a parallel connection to the radiation admittance Y_w . It is noted that the use of an equivalent transverse admittance per unit length Y_{in}/s is based on the assumption that the period s be sufficiently smaller than the guided wavelength of the considered mode, so that a homogenization of the periodic structure can be performed.

As shown in Fig. 1(b), the input admittance of an open-circuited transmission line is

$$Y_{in} = jY_c \tan \left[\frac{2\pi}{\lambda} (l + \Delta l) \right] \quad (5)$$

where $Y_c = 1/Z_c$ is the characteristic admittance of the microstrip line, Δl is the equivalent extension of the microstrip line. The values of Y_c , Δl and λ are given in [15]. With the open-circuited stub loading the MLWA, (4) becomes

$$W_e = W + 2\Delta W - j \frac{h}{\epsilon_{re}} + \frac{240\pi h}{k_0 \epsilon_{re} s} Y_c \tan \left[\frac{2\pi}{\lambda} (l + \Delta l) \right]. \quad (6)$$

Equation (6) shows that the real part of W_e increases when the MLWA is loaded with open-circuited stubs, which results in the increase of the normalized phase constant β_z/β_0 and the radiation angle θ_r will increase accordingly.

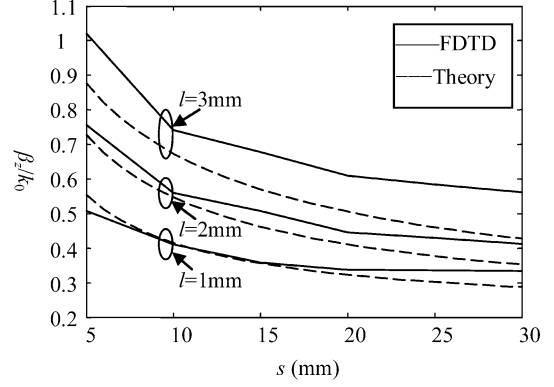
Similarly, as shown in Fig. 1(c), the input admittance of a short-circuited stub is given by

$$Y_{in} = -jY_c \cot \left[\frac{2\pi}{\lambda} (l - 2r) \right] \quad (7)$$

where r is the radius of the pin which is used to short the stub. With these short-circuited stub loading along the MLWA, (4) becomes

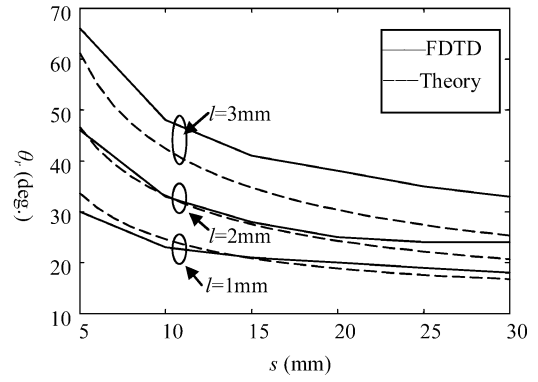
$$W_e = W + 2\Delta W - j \frac{h}{\epsilon_{re}} - \frac{240\pi h}{k_0 \epsilon_{re} s} Y_c \cot \left[\frac{2\pi}{\lambda} (l - 2r) \right]. \quad (8)$$

Equation (8) shows that the real part of W_e decreases when the MLWA is loaded with short-circuited stubs, which results in



$W=11\text{ mm}$, $\epsilon_r=2.2$, $h=0.508\text{ mm}$.

Fig. 5. Normalized phase constant of MLWA with open-circuited stub loading at 8.5 GHz.



$L=120\text{ mm}$, $W=11\text{ mm}$, $\epsilon_r=2.2$, $h=0.508\text{ mm}$.

Fig. 6. Radiation angle of MLWA with open-circuited stub loading at 8.5 GHz.

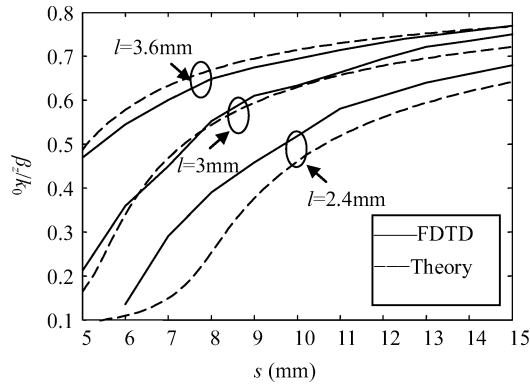
the decrease of the normalized phase constant β_z/β_0 , and the radiation angle θ_r will decrease accordingly.

IV. NUMERICAL CALCULATION AND COMPARISON

Firstly, when the MLWA is loaded with open-circuited stubs, the normalized phase constant β_z/k_0 of MLWA is calculated with (3) and (6) and compared with numerical results as shown in Fig. 5. It shows that the theoretical results agree well with the numerical ones, when $l \leq 2\text{ mm}$ and $s \leq 25\text{ mm}$. But the theoretical results do not fit the numerical ones very well when $l \geq 3\text{ mm}$ and $s > 25\text{ mm}$. That is because the input admittance of the stub calculated by (5) is not very accurate, since the microstrip line is not a perfect transmission line, and that the unwanted susceptance resulting from the connection between the MLWA and stub is not considered. Moreover, when $s > 25\text{ mm}$, the homogenization of the periodic structure is not achieved.

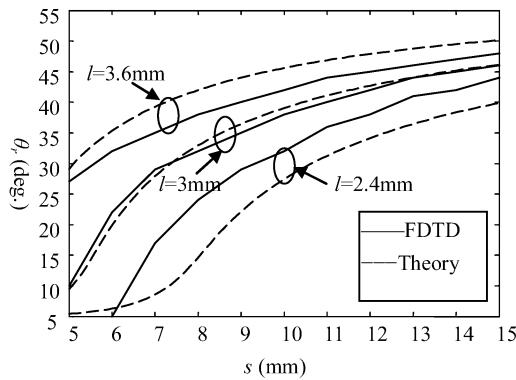
When the phase constant of the MLWA with open-circuited stub loading is calculated, the radiation angle θ_r can be estimated by (1). The theoretically estimated θ_r values are shown in Fig. 6 and compared with the numerical simulation results. It shows that the error of θ_r between the new theory and the FDTD method is within 3° , when $l \leq 2\text{ mm}$, and $s \leq 25\text{ mm}$.

Second, when the MLWA is loaded with short-circuited stubs, the normalized phase constant β_z/k_0 of MLWA is calculated with (3) and (8) and compared with numerical results as shown in Fig. 7. It shows that the theoretical results agree well with the



$W=11\text{ mm}$, $\epsilon_r=2.2$, $h=0.508\text{ mm}$, $r=0.2\text{ mm}$.

Fig. 7. Normalized phase constant of MLWA with short-circuited stub loading at 10.5 GHz.



$L=120\text{ mm}$, $W=11\text{ mm}$, $\epsilon_r=2.2$, $h=0.508\text{ mm}$, $r=0.2\text{ mm}$.

Fig. 8. Radiation angle of MLWA with short-circuited stub loading at 10.5 GHz.

numerical ones, when $3\text{ mm} \leq l \leq 3.6\text{ mm}$. But the theoretical results do not fit the numerical ones very well when $l > 3.6\text{ mm}$ or $l \leq 2.4\text{ mm}$. Because the input admittance approximated by (7) is not very accurate, since the microstrip is not a perfect transmission line and the unwanted susceptance resulting from the connection and from the shorting pin is not taken into consideration.

After the phase constant of the MLWA with short-circuited stub loading is calculated, the radiation angle θ_r can be estimated by (1). The radiation angle θ_r of the MLWA with short-circuited stub loading are estimated theoretically and compared with the numerical simulation results as shown in Fig. 8. It shows that the error of θ_r between the new theory and the FDTD method is within 4° , when $3\text{ mm} \leq l \leq 3.6\text{ mm}$.

V. CONCLUSION

A new theory is provided for analyzing a novel MLWA with shorted stub loading in this letter. This novel MLWA provides

a new way of constructing a fixed-frequency beam steering MLWA. The error of the radiation angle θ_r between the new theory and the FDTD method is within 3° , when $l \leq 2\text{ mm}$, and $s \leq 25\text{ mm}$, for the MLWA with open-circuited stub loading. The error of θ_r between the new theory and the FDTD method is within 4° , when $3\text{ mm} \leq l \leq 3.6\text{ mm}$, for the MLWA with short-circuited stub loading.

After all, the estimation of the radiation angle with this new theory can save a lot of time from numerical analysis. Besides, the shorted stub is considered to be reactive element in the analysis, so we can calculate the radiation angle of MLWA loaded with any reactive element using (1), (3), and (4), only if the input admittance of the reactive element can be estimated. This theory is very convenient for the design of the MLWA with reactive element loading.

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