Chapter 1

TOWARD A CONSISTENT THEORY OF RELATIVISTIC ROTATION

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Abstract

Part 1: Traditional Analysis Conundrums

Although most physicists presume the theoretical basis of relativistically rotating systems is well established, there may be grounds to call the traditional analysis of such systems into question. That analysis is argued to be inconsistent with regard to its prediction for circumferential Lorentz contraction, and via the hypothesis of locality, the postulates of special relativity. It is also contended that the traditional analysis is in violation of the continuous and single valued nature of physical time. It is further submitted to be in disagreement with the empirical finding of Brillet and Hall, the global positioning system satellite data, and a light pulse arrival time analysis of the Sagnac experiment.

Part 2: Resolution of the Conundrums: Differential Geometry and Non-time-orthogonality

It is postulated that physical constraints on time (its continuous and single-valued nature) limit the set of possible synchronization/simultaneity schemes in rotation to one, the "flash from center" scheme. A differential geometry analysis based on this simultaneity postulate is presented in which the rotating frame metric is constrained to be locally non-time-orthogonal (NTO) and due to which, all inconsistencies and disagreements with experiment are resolved. The hypothesis of locality is shown to be invalid for rotation specifically, and generally valid only for non-inertial frames in which the metric can have all null off diagonal space-time components (i.e., time is orthogonal to space.) The analysis approach presented does not contravene traditional relativistic theory for translating systems and makes many (but not all) of the same predictions for rotating systems as does the traditional (time orthogonal) analysis.

Part 3: Experiment and Non-time-orthogonal Analysis

Experiments performed from the 1880s to the present to test special relativity are summarized, and their relevance to NTO analysis is presented. One test, that of Brillet and Hall, appears capable of discerning between the NTO and traditional approaches to relativistic rotation. It yielded a signal predicted by NTO analysis, but not by the traditional approach. Other evidence in favor of the NTO approach may be inherent in the global positioning system data, and the Sagnac experiment.

Keywords: Relativistic rotation, non-time-orthogonal, hypothesis of locality, Brillet and Hall, Sagnac

1. Traditional Analysis Conundrums

1.1 Introduction

Part 1 outlines the traditional approach to relativistic rotation and discusses various apparent inconsistencies associated therewith. Following an analysis of synchronization/simultaneity in rotating frames and seeming traditional approach problems therein, a new postulate is introduced, which will be used in Part 2 to pose an alternative approach to resolving the inconsistencies.

1.2 Relevant Relativity Principles

Special relativity theory (SRT) is restricted to inertial systems and is derived from two symmetry postulates:

1 The speed of light is the same for all inertial observers (it is invariant) and equals c.

2 There is no preferred inertial reference frame. (Velocity is relative, and the laws of nature are covariant, i.e., the same for all inertial observers.)

The first postulate, applied to the one-way speed of light, is equivalent to demanding that Einstein synchronization of clocks holds. In Einstein synchronization, one starts from a first clock at time t_A on that clock and sends a light pulse to a second clock fixed in the same frame as the first. The light pulse is reflected back at the second clock and returns to the first clock at time t_B on the first clock. The time on the second clock is then set such that its reading when the light was reflected would have been $(t_A + t_B)/2$, the time on the first clock half way between the emission and reception times. This ensures the one-way speed of light, measured as the distance traveled between clocks divided by the time difference of the two clocks, is always c.

In recent years, many relativists have come to consider Einstein synchronization merely a convention, or gauge, that affects no measurable quantities[1],[2]. For example, in all such gauge theories of synchronization, the round trip speed of light is c (though the one-way speed of light need not be.) Nevertheless, underlying SRT is the assumption that Einstein synchronization is always one of the possible conventions that makes valid predictions about inertial frames in the physical world.

General relativity is applicable to non-inertial systems and is based on additional postulates, including the equivalence principle and the hypothesis of locality (or sometimes, the "surrogate frames postulate"). The hypothesis of locality stands as a linchpin in the traditional approach to relativistic rotation, and thus, I number it among the postulates of importance to this article.

3 Hypothesis of locality: Locally (i.e., over infinitesimal regions of space and time), neither gravity nor acceleration changes the length of a standard rod or the rate of a standard clock relative to a nearby freely falling (i.e., inertial) standard rod or standard clock instantaneously co-moving with it. See Møller[3], Einstein[4], and Mashoon[5].

Stated another way, a local inertial observer is equivalent to a local co-moving non-inertial observer in all matters having to do with measurements of distance and time. It follows immediately that Einstein synchronization can be carried out locally, and that for such synchronization, the local one-way speed of light measured in a *non-inertial* frame is c. Hence, a Lorentz frame can be used as a local surrogate for the non-inertial frame. This has a basis in differential geometry, in which a curved space is locally flat and can be represented locally by Cartesian coordinates.

Minguzzi[6] and Møller[7], among others, note that the hypothesis of locality is only an assumption. It is, however, an assumption that, historically, has worked very well in a large number of applications. See, for example, the treatment of acceleration by Misner, Thorne, and Wheeler [8] using instantaneous local Lorentz frames.

1.3 The Traditional Approach

The traditional approach to relativistic rotation assumes the hypothesis of locality is a fundamental and universal truth. As done successfully in other, non-rotating, cases, values in local co-moving Lorentz frames are integrated to determine global values for quantities such as distance and time, which would, in principle, be measured with standard meter sticks and clocks by an observer in the rotating frame.

The oft-cited example, first delineated by Einstein[9], is the purported Lorentz contraction of the rim of a rotating disk. (Or alternatively, the circumferential stresses induced in the disk when the rim tries to contract but is restricted from doing so via elastic forces in the disk material.) A local Lorentz frame instantaneously co-moving with a point on the rim, it is argued, exhibits Lorentz contraction of its meter sticks in the direction of the rim tangent, via its velocity, $v = \omega r$. This infinitesimal length contraction is subsequently integrated over all of the local Lorentz frames instantaneously at rest with respect to each successive point along the rim. The result is a number of meter sticks that is greater than $2\pi r$, and thus, the disk surface is concluded to be non-Euclidean, or Riemann curved[10],[11].

1.4 Inconsistency of Circumferential Lorentz Contraction

According to SRT, an observer does not see his own lengths contracting. Only a second observer moving relative to him sees the first observer's length dimension contracted. Hence, from the point of view of the disk observer, her own meter sticks are not contracted[12], and there can be no curvature of the rotating disk surface. The traditional analysis is thus, inconsistent[13].

Consider further the disk observer looking out at the meter sticks at rest in the lab close to the disk's rim. Via the hypothesis of locality (in which she is equivalent to a local co-moving Lorentz observer), she sees the lab meter sticks as having a velocity with respect to her. Hence, by the traditional logic, she sees them as contracted in the circumferential direction. She must therefore conclude that the lab surface is curved. But those of us living in the lab know this is simply not true, and again the analysis is inconsistent.

Although these arguments seem to be rarely considered by traditionalists, when they are brought to their attention, the usual defense is that "the rotating frame is not an inertial frame and thus is different". Yet, the hypothesis of locality, the starting point for the analysis, assumes that they are not different in this regard.

Furthermore, if the non-inertial argument has any validity, then it must imply that the length contraction of the rim is absolute, i.e., both the lab and disk observer agree that the disk meter sticks are contracted. Yet, consider the limit case of low ω, high r, such that $a = \omega^2 r \approx 0$, while $v = \omega r$ is close to the speed of light (the "limit case"). Advocates of the traditional approach contend that, since the limit case observer fixed to the rotating disk rim feels no inertial "force", she becomes, effectively, a Lorentz observer. In this case, each of the lab and disk observers must see the other's meter sticks as contracted and their own as normal. Yet, the non-inertial argument started with the assumption that the disk observer's meter sticks contracted in an absolute way, agreed to by all observers[14].

Conclusion: Length contraction applied via the traditional analysis to rotating systems appears self-contradictory.

1.5 Second SRT Postulate Not Valid in Rotation

Without looking outside, an observer on the rim of a rotating disk can determine her angular velocity ω, using, for example, a Foucault pendulum. She can also use a spring mass system to measure $kx/m = \omega^2 r$, and hence determine r, the distance to the center of rotation. (The Newtonian limit is used to simplify the example. The conclusion is also true for relativistic calculations.)

That is, contrary to the dictate of the second postulate, there are experiments an observer can perform locally from entirely within the rotating frame to determine her speed in an absolute sense. (To be precise, her speed with respect to the inertial frame in which her center of rotation is fixed.) Her velocity is not relative. Both the lab and the rim fixed observers determine the same value for it. With respect to circumferential speed, there is a preferred frame, and both observers agree it is the one where such speed is zero, i.e., the non-rotating lab frame.

Conclusion: The second relativity postulate does not appear to hold for rotating systems

1.6 First SRT Postulate: Thought and Sagnac Experiments

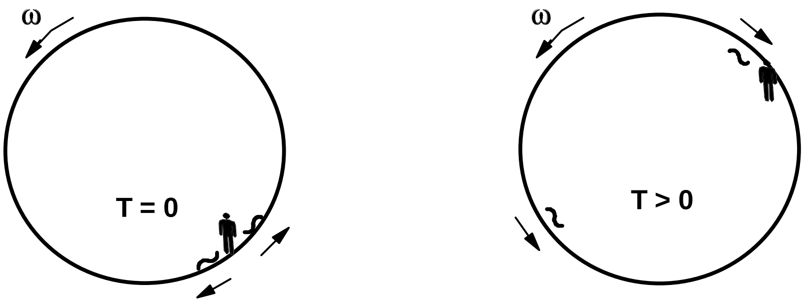

Figure 1.1. Rotating Disk Observer Measuring Light Speed

1.6.1 Thought Experiment. Consider the following thought experiment (see Selleri[15]) involving an observer fixed to the rotating disk of Figure 1 who measures the speed of light.

The observer shown has already laid meter sticks along the rim circumference and determined the distance around that circumference. As part of her experiment, she has also set up a cylindrical mirror, reflecting side facing inward, all around the circumference. She takes a clock with her and anchors herself to one spot on the disk rim. When her clock reads time $T = 0$ (left side of Figure 1) he shines two short pulses of light tangent to the rim in opposite directions. The mirror will cause these light pulses to travel circular paths around the rim, one clockwise (cw) and one counterclockwise (ccw).

From the lab, we see the cw and ccw light pulses having the same speed c. However, as the pulses travel around the rim, the rim and the observer fixed to it move as well. Hence, a short time later, as illustrated in the right side of Figure 1, the cw light pulse has returned to the observer, whereas the ccw pulse has yet to do so. A little later (not shown) the ccw pulse will have caught up to the observer.

For the observer, from her perspective on the disk, both light rays travel the same number of meters around the circumference. But her experience and her clock readings tell her that the cw pulse took less time to travel the same distance around the circumference than the ccw pulse.

She can only conclude that, from her point of view, the cw pulse traveled faster than the ccw pulse. Hence, the speed of light as measured on the rotating disk is anisotropic and different from that measured in the lab. Thus, one could conclude that the first relativity postulate, in the context of the hypothesis of locality, is violated.

Arguments against this conclusion are rooted in two interrelated concepts: i) purported differences between the global (as measured in the above thought experiment) and local, physical speeds of light[16],[17],[18], and ii) the synchronization/simultaneity employed[19],[20]. The author has extensive remarks on this in a subsequent section, but for now, submits that the appropriate synchronization scheme comprises the following.

Consider the ccw light pulse and the time difference on the clock held by the observer in Figure 1 between the emission (assume initial clock time is $t_A = 0$) and reception (clock time t_B) events. Employ the synchronization method of Section 1.2, only instead of using a back and forth round trip for light (Einstein synchronization), use a circular round trip. That is, the time on the clock half way along the round trip ccw path (at 180° of the disk here), at the instant the ccw light pulse was there, should be $(t_A + t_B)/2 = t_B/2$. With this time, the ccw speed of light will be the same as that computed for the round trip, i.e., it will be less than c. Now synchronize the clock at 90° the same way. Assume its setting at the instant the light pulse passed was half that on the clock at 180° when the light passed that clock (or equivalently, $1/4$ of t_B.) Doing this, one again finds the ccw speed of light to be the same value, which is $< c$. Repeat over smaller intervals until, in the limit, one finds the local speed of light to be the same, and therefore not equal to c.

The entire procedure can be repeated for the cw light pulse, and one will find the clocks at each location done via the cw and ccw methods are synchronized, i.e., they are the same clocks. One will also find that the local speeds of light are anisotropic and equal to the same values as the global ones determined using a single clock at the emission/reception point.

Conclusion: While it may appear that the local speed of light, being anisotropic, violates the first relativity postulate, there is still the possibility that Einstein synchronization may be valid locally (as one of the possible local synchronization schemes). This would mean that for such synchronization, the local speed of light would be c, and one could get correct results using the hypothesis of locality and local Lorentz frame analyses.

Challenge to traditionalists: The author does not believe this and challenges advocates of the traditional approach to begin with the assumption of local isotropic light speed, and without looking outside of the rotating frame, kinematically derive the result that the cw light pulse arrives back at the emission point before the ccw one.

1.6.2 The Sagnac Experiment. In 1913, G. Sagnac[21] carried out a now famous experiment, similar in many ways to the thought experiment of Figure 1. On a rotating platform, he split light from a single source into cw and ccw rays that traveled identical paths in opposite directions around the platform. He combined the returning rays to form a visible interference pattern, and found that the fringes shifted as the speed of rotation changed. A number of others[22],[23] subsequently performed the same test with the same results.

If the speed of light were locally invariant and always equal to c, then speeding up or slowing of the rotation rate of the platform should not change the location of the fringes. However, the fringes do change with speed and once again we have a test (Sagnac) whereby we can determine a preferred frame, in seeming violation of the second relativity postulate and the hypothesis of locality.

Putative explanations for this in the context of the traditional approach hinge, once again, on synchronization/simultaneity and global vs. local arguments. These are addressed in the following section.

I do contend that the thought experiment of Figure 1 makes it clear that any explanation for the Sagnac experiment, from the point of view of the disk reference frame, must account for different *arrival times* for the cw and ccw light pulses. Analyses based on Doppler shifts[24] or DeBroglie momentum/wave length[25] changes are simply not sufficient to explain this.

The calculation of this arrival time difference, derived from the lab frame, is well known and is repeated for reference in the Appendix.

1.7 Synchronization/Simultaneity in the Traditional Approach

1.7.1 The Traditional Approach "Time Gap". Consider the non-rotating (lab) frame as K; the rotating (disk) frame as k. Figure 2 depicts inertial measuring rods in inertial frames K_1 to K_8 with speeds ωr instantaneously at rest with respect to eight points on the rotating disk rim as shown. For practical reasons, only eight finite length rods are shown, and one can consider them as a symbolic representation of an infinite number of such rods of infinitesimal length. A and B are *events* located in space at the endpoints of the K_1 rod which are simultaneous as seen from K_1; B and C are events located in space at the endpoints of the K_2 rod which are simultaneous in K_2; and so on for the other events C to J. A,B, ...J can be envisioned as flashes of light emitted by bulbs situated equidistantly around the disk rim.

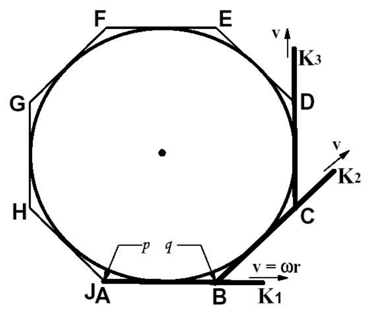

Figure 1.2. Inertial Co-Moving Frame

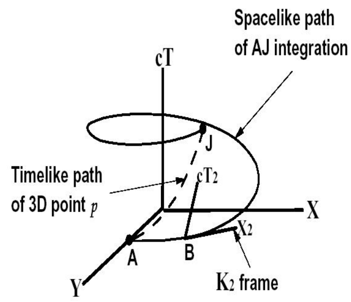

Figure 1.3. Co-Moving Frames Integration Path

p is a spatial (three-dimensional) point fixed to the disk at which both A and J occur. q is the spatial point on the disk at which B occurs. In principle, A, B, ... J, as well as p and q are located on the disk rim though they may not look so in Figure 2, since the co-moving rods shown are not infinitesimal in length.

In the traditional analysis, the hypothesis of locality is invoked to claim that times and distances measured by standard measuring rods and clocks in the local co-moving inertial frames are identical to those riding with the disk.

Note that although events A and B are simultaneous as seen from K_1, they are not simultaneous as seen in K (via SRT for two inertial frames in relative motion). As seen from K, A occurs before B. Similarly, B occurs before C, and so on around the rim. If the events are light flashes, a ground based observer looking down on the disk would see the A flash, then B, then C, etc. Hence we conclude that as seen from K, A occurs before J even though A and J are both located at the same 3D point p fixed to the rim. As seen from K, during the time interval between events A and J, the disk rotates, and hence the point p moves. (As an aside, Figure 2 can now be seen to be merely symbolic since events A to J would not in actuality be seen from K to occur at the locations shown in Figure 2. That is, by the time the K observer sees the B flash, the disk has rotated a little. It rotates a little more before he sees the C flash, etc.)

According to the traditional treatment of the rotating disk, one then uses the K_i rods and integrates (adds the rod lengths) along the path AB ...J, moving sequentially from co-moving inertial frame to co-moving inertial frame. This path is represented by the solid line in Figure 3, and one can visualize small Minkowski coordinate frames situated at every point along the curve AJ (see K_2 in Fig. 3) with integration taking place along a series of spatial axes (such as X_2 in Fig. 3). By doing this one arrives at a length for AJ, the presumed circumference of a disk of radius r, of

$$\mathrm{AJ} = \frac{2\pi r}{\sqrt{1-\omega^2 r^2/c^2}} > \text{ Circumference in lab,} \tag{1.1}$$

and thus, the disk surface is concluded to be non-Euclidean (Riemann curved.)

But consider that since point p moves along a timelike path as seen from K (see dotted line in Fig. 3), a time difference between events A at 0^o and J at 360^o must therefore exist as measured by a clock attached to point p. To continue from J to A requires a jump in time, and thus, the traditional analysis approach leads to a discontinuity in time (or alternatively, multi-valued time), a seemingly impossible physical situation. Further, as noted by Weber[26], this means that if light rays are sent 360^o around the rim to synchronize the clock at J with that at A, then the two clocks (which are really one and the same clock) are not in synchronization. That is, each clock on the disk is out of synchronization with itself.

Still further, according to the traditional analysis, time all along the path AJ is fixed. Thus, by that analysis, which depends on the locality hypothesis and integration of values (time in this case) from local frame to local frame, A and J must be simultaneous. But they are not.

1.7.2 Traditional Approach to Resolve the "Time Gap".

In recent years, this problem has been treated as if this "time gap" were a mathematical artifact, and approaches labeled "desynchronization" [27],[28],[29] and "discontinuity in synchronization"[30],[31] have been proposed that entail multiple clocks at a given event. These approaches seem motivated by the gauge theory of synchronization philosophy that time settings on clocks are inherently arbitrary.

Furthermore, the time gap is often said to be identical in nature to traveling at constant radius in a polar coordinate system. The ϕ value is discontinuous at 360^o. Similar logic applies for time, with the International Date Line for the time zone settings on the earth. If one starts at that line and proceeds 360^o around the earth, one returns to find one must jump a day in order to re-establish one's clock/calendar correctly.

1.7.3 Arguments for Physical Interpretation of the "Time Gap".

The gauge synchronization philosophy champions innumerable, equally valid, synchronization schemes. Yet, within any one of those schemes, time is single valued and continuous, and clocks are all in synchronization with themselves. For a given synchronization method, each event within a given frame has a single time associated with it.

In the desynchronization approaches, a given event in a given rotating frame, for a given synchronization method, can have any number of possible times on it. For example, the clock at point p in Fig. 2 has one time on it at event A. If one Einstein synchronizes the clock at 360^o (i.e., the same clock at p) with ccw light rays, one gets another time setting. Thus, one has a choice of which of two times one prefers for any given event at point p. If, on the other hand, one synchronizes the clock at 360^o via cw light rays, one gets yet another setting, and a third possible time to choose for any given event. Consider yet another path in which the light ray goes radially inward 1 meter, then 360^o around, then radially outward 1 meter. One then gets yet another setting for the clock at p. Since there are an infinite number of possible paths by which one could synchronize the clock at p, there are an infinite number of possible times for each event at p. (This does not happen in translation. Any possible path for the light rays results in the same unique setting, for a given synchronization scheme, on each clock in the frame.)

This plethora of possible settings for the same clock results from insisting on "desynchronization" of clocks in order to keep the speed of light locally c everywhere. And thus, one is in the position of choosing whichever value for time one needs in a given experiment in order to get the answer one insists one must have (i.e., invariant, isotropic local light speed.) One can only then ask if this is really physics or not. Can an infinite number of possible readings on a single clock at a single event for a single method of synchronization be anything other than meaningless?

The polar coordinate analogy, I believe, confuses physical discontinuity with coordinate discontinuity. In 2D, place a green X at 0^o, travel 360^o at constant radius, and then place a red X. The red and green marks coincide in space. There is no discontinuity in space between them, although there is a discontinuity in the coordinate ϕ.

Flash a green light on the equator at the International Dateline, then trace a path once around the equator along which no time passes. If you flash a red light at the end of that path, the red and green flashes are coincident in space and time. There is no physical discontinuity, although your time zone clocks show a coordinate discontinuity.

In Fig. 3, flash a green light at event A. Travel 360^o on the disk along the space-time path AJ (along which no time passes according to the traditional analysis), then flash a red light at event J. The two lights are not coincident. There is real world space-time gap between them, and they exist at different points in 4D. The discontinuity is physical, not merely coordinate.

Peres was aware of this time discontinuity, calling it a *"heavy price which we are paying to make the [circumferential] velocity of light ... equal to c"*[32]. Dieks[33] noted that though arbitrary in certain senses, time in relativity must be *"directly linked to undoubtedly real physical processes"*. This author agrees.

1.7.4 The Only Physically Possible Synchronization/Simultaneity.

There are potential choices for synchronization/simultaneity in the rotating frame other than Einstein's. The traditional one with local Einstein synchronization around the disk rim is based on the Lorentz transformation from the lab to the local co-moving inertial frame, i.e.

$$cdT_i = \frac{1}{\sqrt{1 - v^2/c^2}}(cdT - \frac{v}{c}dX) = \gamma(cdT - \frac{v}{c}dX) \tag{1.2}$$

where $v = \omega r$, dT is the time interval in the lab, dX is the space interval in the lab along the disk rim, dT_i is the time interval in the local co-moving inertial frame, which we presume, by the locality hypothesis, equals the time interval on the disk. We could just as well have chosen [34]

$$cdT_i = \gamma(cdT - \kappa dX) \tag{1.3}$$

where κ could have any value other than v/c.

However, for any $\kappa \neq 0$, we would again have a time discontinuity, and all the issues with multiple event times and clocks being out of synchronization with themselves of the prior section.

I suggest that, prior to all else, any theory of rotation must be compliant with the physical world constraint that time be continuous and single valued (within a given frame, and for a given synchronization scheme.) That is only possible for a synchronization/simultaneity scheme where $\kappa = 0$. For this scheme, events in the lab that are simultaneous (i.e., have $dT = 0$ between them) are also simultaneous in the rotating frame (have $dt = 0$).

Postulate: Any synchronization/simultaneity scheme for the rotating frame for which that frame and the lab do not share common simultaneity results in a physical time discontinuity and is thus unacceptable on physical grounds.

The traditional approach to rotation is at odds with this postulate.

1.8 Experiment and the Traditional Approach

In Part 3, virtually all of the experiments that have been carried out to verify SRT are reviewed. One of these, the Michelson-Morley type experiment performed by Brillet and Hall[35], found a persistent, anomalous, non-null signal at the 10^{-13} level, which is not predicted by SRT. The approach to relativistic rotation of Part 2, which is based on the above postulate, predicts this signal, and otherwise, makes the same predictions as the traditional approach for the remaining tests.

Furthermore, as a result of studies on the Global Positioning System (GPS) data for the rotating earth, recognized world leading GPS expert Neil Ashby states

"Now consider a process in which observers in the rotating frame attempt to use Einstein synchronization..... Simple minded use of Einstein synchronization in the rotating frame ... thus leads to a significant error".[36]

He also recently noted in *Physics Today*,

" .. the principle of the constancy of c [the speed of light] cannot be applied in a rotating reference frame .." [37].

1.9 Summary of Part 1

Thought experiments, actual experiments, and the physical nature of the space-time continuum appear discordant with the traditional approach to relativistic rotation.

2. Resolution of the Conundrums: Differential Geometry and Non-time-orthogonality

".. a good part of science is distinguishing between useful crazy ideas and those that are just plain nutty."

Princeton University Press advertisement for the book "Nine Crazy Ideas in Science"

2.1 Introduction

Part 2 poses an alternative approach to relativistic rotation that resolves the inconsistencies, and as will be seen in Part 3, appears to have better agreement with experiment than the traditional approach. There are two fundamental steps to the alternative approach.

1 Postulate that, in accord with physical world logic as presented in Part 1, simultaneity/synchronization in the rotating frame can only be such that time in that frame is continuous and single valued.

2 Apply differential geometry and note resulting predictions.

Before beginning the analysis, relevant background material from differential geometry is presented in Section 2.2.

2.2 Physical vs. Coordinate Components

If one has coordinate components, found from generalized coordinate tensor analysis, for some quantity, such as stress or velocity, one needs to be able to translate those into the values measured in experiment. For some inexplicable reason, the method for doing this is not typically taught in general relativity (GR) texts/classes, so it is reviewed here. (Note that often in GR, one seeks invariants like $d\tau$, ds, etc., which are the same in any coordinate system, and in such cases, this issue does not arise. The issue does arise with vectors/tensors, whose coordinate components vary from coordinate system to coordinate system.)

The measured value for a given vector component, unlike the coordinate component, is unique within a given reference frame. In differential geometry (tensor analysis), that measured value is called the "physical component".

Many tensor analysis texts show how to find physical components from coordinate components[38]. A number of continuum mechanics texts do as well[39]. The only GR text known to the present author that mentions physical components is Misner, Thorne, and Wheeler[40]. Those authors use the procedure to be described below, but do not derive it[41]. The present author has written an introductory article on this, oriented for students, that may be found at the Los Alamos web site[42]. The following is excerpted in part from that article.

The displacement vector $d\mathbf{x}$ between two points in a 2D Cartesian coordinate system is

$$d\mathbf{x} = dX^1\hat{\mathbf{e}}_1 + dX^2\hat{\mathbf{e}}_2 \tag{1.4}$$

where the $\hat{\mathbf{e}}_i$ are unit basis vectors and dX^i are physical components (i.e., the values one would measure with meter sticks). For the same vector $d\mathbf{x}$ expressed in a different, generalized, coordinate system we have different coordinate components $dx^i \neq dX^i$ (dx^i do not represent values measured with meter sticks), but a similar expression

$$d\mathbf{x} = dx^1\mathbf{e}_1 + dx^2\mathbf{e}_2, \tag{1.5}$$

where the generalized basis vectors $\mathbf{e}_i$ point in the same directions as the corresponding unit basis vectors $\hat{\mathbf{e}}_i$, but are not equal to them. Hence, for $\hat{\mathbf{e}}_i$, we have

$$\hat{\mathbf{e}}_i = \frac{\mathbf{e}_i}{|\mathbf{e}_i|} = \frac{\mathbf{e}_i}{\sqrt{\mathbf{e}_{\underline{i}} \cdot \mathbf{e}_{\underline{i}}}} = \frac{\mathbf{e}_i}{\sqrt{g_{\underline{ii}}}} \tag{1.6}$$

where underlining implies no summation.

Substituting (1.6) into (1.4) and equating with (1.5), one obtains

$$dX^1 = \sqrt{g_{11}}dx^1 \qquad dX^2 = \sqrt{g_{22}}dx^2, \tag{1.7}$$

which is the relationship between displacement physical (measured with instruments) and coordinate (mathematical value only) components.

Consider a more general case of an arbitrary vector $\mathbf{v}$

$$\mathbf{v} = v^1\mathbf{e}_1 + v^2\mathbf{e}_2 = v^{\hat{1}}\hat{\mathbf{e}}_1 + v^{\hat{2}}\hat{\mathbf{e}}_2 \tag{1.8}$$

where, $\mathbf{e}_1$ and $\mathbf{e}_2$ here do not, in general, have to be orthogonal, $\mathbf{e}_i$ and $\hat{\mathbf{e}}_i$ point in the same direction for each index i, and carets over component indices indicate physical components. Substituting (1.6) into (1.8), one readily obtains

$$v^{\hat{i}} = \sqrt{g_{\underline{ii}}}v^i, \tag{1.9}$$

which we have shown here to be *valid in both orthogonal and non-orthogonal systems.*

As a further aid to those readers familiar with anholonomic coordinates (which are associated with non-coordinate basis vectors superimposed on a generalized coordinate grid), physical components are special case anholonomic components for which the non-coordinate basis vectors have unit length.

It is important to recognize that anholonomic components do not transform as true vector components. So one can not simply use physical components in tensor analysis as if they were. Typically, one starts with physical components as input to a problem. These are converted to coordinate components, and the appropriate tensor analysis carried out to get an answer in terms of coordinate components. One then converts these coordinate components into physical components as a last step, in order to compare with values measured with instruments in the real world.

As a basis vector is derived from infinitesimals (derivative at a point), one sees (1.9) is valid locally in curved, as well as flat, spaces, and can be extrapolated to 4D general relativistic applications. So, very generally, for a 4D vector v^μ and a metric signature (-,+,+,+)

$$v^{\hat{i}} = \sqrt{g_{\underline{ii}}}v^i \qquad v^{\hat{0}} = \sqrt{-g_{00}}v^0, \tag{1.10}$$

where Roman sub and superscripts refer solely to spatial components (i.e. i = 1,2,3.)

2.3 Alternative Analysis Approach

We begin with the simultaneity postulate of Section 1.7.4, repeated below for convenience.

Postulate: Any synchronization/simultaneity scheme for the rotating frame, for which that frame and the lab do not share common simultaneity, results in a physical time discontinuity and is thus unacceptable on physical grounds.

2.3.1 Disk Transformation and Metric. As will be discussed, the global transformation from the lab to the rotating frame apparently first used by Langevin[43] to find a suitable metric for the rotating frame incorporates the above postulate. This transformation is used in the following analysis, which parallels that of Klauber[44]. The correctness of the transformation can be judged by whether the predictions made by using it match experiment.

This transformation, between the non-rotating (lab, upper case symbols) frame to a rotating (lower case) frame, is

$$\begin{aligned} cT &= ct \\ R &= r \\ \Phi &= \phi + \omega t \\ Z &= z\,. \end{aligned} \tag{1.11}$$

ω is the angular velocity of the rotating frame as seen from the lab, and cylindrical spatial coordinates are used. The coordinate time t for the rotating system equals the proper time of a standard clock located in the lab. Substituting the differential form of (1.11) into the line element in the lab frame

$$ds^2 = -c^2dT^2 + dR^2 + R^2d\Phi^2 + dZ^2 \tag{1.12}$$

results in the line element for the rotating frame

$$ds^2 = -c^2(1-\tfrac{r^2\omega^2}{c^2})dt^2 + dr^2 + r^2d\phi^2 + 2r^2\omega d\phi dt + dz^2 = g_{\alpha\beta}dx^\alpha dx^\beta. \tag{1.13}$$

Note that the metric in (1.13) is not diagonal, since $g_{\phi t} \neq 0$, and this implies that time is not orthogonal to space (i.e., a non-time-orthogonal, or NTO, frame.)

2.3.2 Time on the Disk. Time on a standard clock at a fixed 3D location on the rotating disk, found by taking $ds^2 = -c^2d\tau$ and $dr = d\phi = dz = 0$ in (1.13), is

$$d\tau = \sqrt{1 - r^2\omega^2/c^2}dt = \sqrt{-g_{tt}}dt, \tag{1.14}$$

This varies with radial position r. At the axis of rotation (where $r = 0$), the standard clock agrees with the clock in the lab. At other locations, standard clock time is diluted by the Lorentz factor, as in traditional SRT. The coordinate time everywhere on the disk is t, and that equals the time T in the lab.

The time difference between two events at two locations (each having its own clock) on the disk, in coordinate components, is dt. The corresponding physical time difference is

$$dt_{phys} = d\hat{t} = \sqrt{-g_{tt}}dt = \sqrt{1 - r^2\omega^2/c^2}dt. \tag{1.15}$$

If the two locations happen to be one and the same location, one obviously gets (1.14).

Note that two events seen as simultaneous in the lab have $dT = 0$ between them. From the first line of (1.11) and (1.15), the same two events must also have $dt_{phys} = 0$, and thus they are also seen as simultaneous on the disk. This statement is true for all standard (physical) clocks on the disk. Though the standard clocks at different radii thereon run at different rates, and thus can not be synchronized, they *can* share common simultaneity. The lab shares this common simultaneity with all of the disk clocks, and thus our postulate above holds for transformation (1.11), and the resulting (NTO) metric of (1.13).

Note further, that the simultaneity chosen here is equivalent in the physical world to what is sometimes called "flash from center" simultaneity (or synchronization if one is confined to clocks at fixed radius). In that scheme, one imagines a flash of light on the axis of rotation whose wave front propagates outwardly in all radial directions. Events when the wave front impacts individual points along a given circumference are considered simultaneous.

It is significant that the "flash from center" synchronization is the same as that proposed (via other logic) near the end of Section 1.6.1.

2.3.3 Local Speed of Light on the Disk. For light $ds^2 = 0$. Inserting this into (1.13), taking $dr{=}dz{=}0$, and using the quadratic equation formula, one obtains a local coordinate velocity (generalized coordinate spatial grid units per coordinate time unit) in the circumferential direction

$$v_{light,coord,circum} = \frac{d\phi}{dt} = -\omega \pm \frac{c}{r}, \tag{1.16}$$

where the sign before the last term depends on the circumferential direction (cw or ccw) of travel of the light ray. The local physical velocity (the value one would measure in experiment using standard meter sticks and clocks in units of meters per second) is found from this to be

$$v_{light,phys,circum} = \frac{\sqrt{g_{\phi\phi}}d\phi}{\sqrt{-g_{tt}}dt} = \frac{-\,r\omega\ \pm\ c}{\sqrt{1 - \frac{\omega^2 r^2}{c^2}}} = \frac{-\,v\ \pm\ c}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{1.17}$$

Note that for this approach, the local physical speed of light in rotating frames is not invariant or isotropic, and that this lack of invariance/isotropy depends on ω, the angular velocity seen from the lab. Note particularly that this result is a direct consequence of the NTO nature of the metric in (1.13). If ω=0, local physical (measured) light speed is isotropic and invariant, the metric is diagonal, and time is orthogonal to space.

I thus call this alternative analysis method, the NTO approach to relativistic rotation.

2.4 Implications of NTO Approach

2.4.1 Hypothesis of Locality. Local physical light speed in the rotating frame, according to the NTO approach, is not equal to c. Yet, in a local, co-moving, Lorentz frame, which via the hypothesis of locality is equivalent locally to the non-inertial frame, the physical speed of light is always c. Thus, a local co-moving Lorentz frame is *not* equivalent locally to the rotating frame, and the hypothesis of locality is not valid for such frames. One does *not* measure the same values for velocity, and hence by implication for time and space, in the two frames.

This is a direct result of the simultaneity postulate, required to keep time in the rotating frame continuous and single-valued. That requirement results in a rotating frame that can only be NTO, i.e., it can only have a metric with off diagonal terms in the metric.

I submit that the hypothesis of locality remains true only for those non-inertial frames in which it is possible for the metric to have all null off-diagonal space-time components. This set of frames comprises the vast majority of problems encountered in GR. Rotation is a critical exception.

2.4.2 Absolute Nature of Simultaneity in Rotation. In NTO analysis, simultaneity/synchronization on the rotating disk, unlike that in the gauge theory of synchronization, is unique (absolute.) The gauge theory validity, it is submitted, is restricted to translating frames and does not apply to rotation. This is not unlike other differences between rotation and translation. Velocity in rotation, for example, has an absolute quality, whereas in translation it does not. There is a preferred frame in rotation, upon which everyone agrees (the frame with no Coriolis effect, for example); in translation, there is no such frame.

2.4.3 Lorentz Contraction Revisited. To determine Lorentz contraction of meter sticks, we merely need to compare physical length in the circumferential direction in both the lab and rotating frames, i.e., look at the physical component for $d\Phi$ and $d\phi$. This is equivalent to finding the proper length when $dT = 0$ in the first frame (lab here), and $dt = 0$ in the second frame (disk here), which is what one does in SRT.

The distance between two points along the circumference in the lab in meter sticks is

$$d\Phi_{phys} = d\hat{\Phi} = \sqrt{g_{\Phi\Phi}}d\Phi = Rd\Phi = R(\Phi_2 - \Phi_1), \tag{1.18}$$

which is not surprising, and which (for $dR = dT = dZ = 0$) equals ds. (1.18) represents the number of meter sticks between points 1 and 2 in the lab. Now, consider two 3D points on the disk located instantaneously at the same place as points 1 and 2 in (1.18). We ask, how many meter sticks span that distance as measured on the disk? That distance between points 1 and 2 in meter sticks is

$$d\phi_{phys} = d\hat{\phi} = \sqrt{g_{\phi\phi}}d\phi = rd\phi = r(\phi_2 - \phi_1). \tag{1.19}$$

According to (1.11), $\phi_1 = \Phi_1 - \omega t_1$ and $\phi_2 = \Phi_2 - \omega t_2$. Since, $r = R$ and $dt = t_2 - t_1 = 0$, (1.18) and (1.19) are equal. The disk observer sees the same number of meter sticks between two points as the lab observer does between those points, and hence, there is no Lorentz contraction.

Note that we would need a metric component $g_{\phi\phi} \neq r^2$ in the rotating frame to have Lorentz contraction. The postulate of simultaneity/time continuity leads to the metric of (1.13), which is NTO, and which has $g_{\phi\phi} = r^2$.

The Lorentz contraction issue is treated more extensively, and with graphical illustrations, in Klauber[44]. The limit case for NTO analysis is also discussed therein though it is treated at great length in Klauber [45], and found to be free of inconsistencies.

2.4.4 Sagnac and Thought Experiments. A complete and general derivation of the Sagnac result from the rotating frame using NTO analysis can be found in Klauber[46]. Shown below is the simpler derivation for a circumferential light path whose center is the axis of rotation. Note that different speeds for light in the cw and ccw directions is inherent in the NTO approach, and thus that approach is completely consonant with the thought experiment of Part 1, Section 1.6.1.

The difference in time measured on a ccw rotating disk between two pulses of light traveling opposite directions along a circumferential arc of length dl is

$$dt_{phys} = \frac{dl}{v_-} - \frac{dl}{v_+}, \tag{1.20}$$

where v_+ is the speed for the cw light ray and v_- is the speed for the ccw ray. Using (1.17) this becomes

$$dt_{phys} = \frac{dl\sqrt{1-v^2/c^2}}{c-v} - \frac{dl\sqrt{1-v^2/c^2}}{c+v} = \frac{v}{c^2}\frac{2dl}{\sqrt{1-v^2/c^2}}. \qquad (1.21)$$

By integrating the RHS of (1.21) from 0 to $2\pi r$ (recall there is no Lorentz contraction), the LHS becomes the time difference on the clock fixed at the emission/reception point,

$$\Delta t_{phys} = \frac{\omega r}{c^2}\frac{2(2\pi r)}{\sqrt{1-\omega^2 r^2/c^2}} = \frac{4\omega A}{c^2\sqrt{1-\omega^2 r^2/c^2}}, \qquad (1.22)$$

which agrees with the derivation from the lab frame of the Appendix.

2.4.5 Brillet and Hall. The Brillet and Hall[35] experiment is described in Part 3. It remains to this day the only test of sufficient accuracy to detect any non-null Michelson-Morley (MM) effect due to the surface speed of the earth rotating about its axis. Brillet and Hall found null signals for the solar and galactic orbit speeds. However, they noted a persistent non-null signal at 2 X 10^{-13}, which had fixed phase in the lab frame.

This signal is not predicted by traditional SRT, which insists on local Lorentz invariance for light speed, and was thus simply deemed "spurious" without further explanation. However, this signal is predicted by NTO analysis due to the earth surface speed. (See Klauber[47].)

2.4.6 Gravitational Orbit vs. True Rotation. One could ask why any test should get a null signal for the solar and galactic orbital velocities, but a non-null signal for the earth surface speed from its own rotation.

The answer is that a body in gravitational orbit is in free fall, and is therefore Lorentzian. No centrifugal "force" is felt, and no Foucault pendulum moves, as a result of revolution in orbit. There is no experimental means by which one could determine (without looking outside at the stars) one's rate of revolution in orbit. Hence, you can not determine any absolute circumferential speed, and the second postulate of relativity holds. Related logic[48] leads to the conclusion that the speed of light on such a body is invariant and equal to c as well.

Thus, the usual form of relativity should hold for gravitational orbits and one should expect a null Michelson-Morley result for orbital speeds, which is just what is measured. However, one *can* use instruments to determine the speed of the earth's surface about its axis, and therefore we should expect that relativity theory will not hold in precisely the same form for that case. It is submitted that the Brillet and Hall result justifies that expectation.

This subject is treated in depth in Klauber[48].

2.4.7 NTO vs. Selleri Transformations. In treating rotation, Selleri[49] uses the same simultaneity as the lab (though he advocates an "absolute" simultaneity that pervades translation as well.) He finds anisotropic one-way light speed on a rotating disk as

$$v_{light,phys,circum,Selleri} = \frac{-\omega r \pm c}{1 - \omega^2 r^2/c^2} \tag{1.23}$$

for the cw and ccw speeds of light along the circumference. Comparing this with the NTO relation of (1.17), one finds the two differ by a factor of $1/\sqrt{1 - \omega^2 r^2/c^2}$.

Selleri shows that his relation (1.23) results in a circular round trip speed for light (one way around the rim) that agrees with (the first order) Sagnac experimental results. However, for a back and forth round trip for light along the same path, he shows his relation results in a round trip speed of precisely c. Thus, he predicts a null result for any Michelson-Morley type experiment.

NTO analysis on the other hand, due to the Lorentz factor difference from (1.23), predicts a back and forth round trip speed for light as not equal to c. Therefore, it predicts a non-null result for MM experiments (which are sensitive enough to detect effects from the earth surface speed due to its own rotation.)

This difference can be attributed to the lack, in the NTO approach, of circumferential Lorentz contraction, as opposed to the inclusion of such contraction in the Selleri approach. Given Lorentz contraction, light rays will travel a greater number of meter sticks, and thus speed will be increased by the magnitude of that contraction. This is the difference between (1.23) and (1.17).

2.4.8 Co-moving vs. Disk-fixed Observers. It should be clearly noted that in the NTO approach, the rotating disk fixed observer and the local co-moving Lorentz observer are not equivalent. They do not, for example, see the lab meter sticks as having the same length. This is in accord with earlier statements regarding the invalidity of the hypothesis of locality for rotating frames.

From another perspective, it could be claimed that the two observers are not truly co-moving, as the disk observer at r is rotating (at ω), whereas the local Lorentz observer is not.

2.5 Summary of Part 2

By adopting

1) the postulate that time in a rotating frame must be continuous and single valued (each clock must be in synchronization with itself), and

2) the specific transformation of form (1.11) that incorporates that postulate,

one can develop an NTO theory for rotation that resolves all conundrums of Part 1, and in which the physical speed of light is constrained to be locally anisotropic. One finds agreement with experiment, including the Brillet and Hall test result, which is not predicted by the traditional approach to relativistic rotation.

One also finds the hypothesis of locality can only be true for non-inertial frames in which the metric can be expressed in diagonal form and still maintain continuity in time. In rotation, this is not true, and the local co-moving observer does not see events (in particular, meter stick lengths) in the same way as the disk-fixed observer.

3. Experiment and Non-time-orthogonal Analysis

3.1 Introduction

Part 3 reviews the experiments that have been performed to test special relativity, and implicitly therein, the hypothesis of locality and the traditional approach to relativistic rotation. Results of these experiments are examined in order to compare the predictive capacity of the NTO and traditional analysis approaches.

3.2 The Experiments

Table 1 is an extensive list of experiments performed since 1887 capable of evaluating at least one aspect of SRT. Particular experiments are referred to herein via the symbol in the first column. A terse description of each is given in column two, with the year and author citations in column three. Note the acronym SRT implies both special relativity theory and the traditional approach to rotation. Column four briefly summarizes how the NTO effect in a given experiment compares with the traditional approach effect. The last two columns compare the predictions of NTO and the traditional approach (Trad) for the given experiment. For a summary of JPL, Mössbauer, TPA, and GPA, see Will[50]. For a summary of Hughes-Drever, BH, NBS, UWash, and Mössbauer see Haugan and Will[51].

Three experiments known to the author are not included in the table because their results were contrary to both SRT and NTO theories. What these results mean is subject to debate, though most physicists who are aware of them believe they must be in error. The earliest of these was by Miller[52], a highly respected colleague of Michelson. He repeated the Michelson-Morley test four times over many years, with various equipment in various places, and much of the work was done jointly with Morley. The other experiments reporting results contrary to SRT were by Silvertooth[53] and Marinov[54]. In any event, these experiments do not discern between the Trad and NTO approaches, and are referenced here for completeness.

3.3 The Comparisons

Both the traditional and NTO approaches predict time dilation, and experiments measuring this, such as PartAcc (see table), would, for the most part, provide no capability of differentiating between approaches. Also, Doppler shift effects tend to be the same in NTO and Trad, though, for certitude, each experiment comprising Doppler measurement needs to be evaluated on its own.

Tests of the speed of light itself, such as MM, should be more directly indicative. These must, however, have i) sufficient accuracy to detect any effect from the relatively low earth surface speed about its axis, and ii) apparatus that turns with respect to the earth surface. The MM, Post MM, and Joos tests, for example, lacked the first of these. The JPL and CORE experiments lacked the second. The LFV test did not meet either criterion.

For some tests, there is uncertainty. For example, in the ODM experiment, rotation of the apparatus yielded a persistent $\sim$1.5 km/sec variation, which was attributed to the earth's magnetic field. This would, however, mask any NTO effect (at $\sim$0.35 km/sec), and yield uncertainty as to whether Trad predicts the result or not. In the Hughes-Drever test, Doppler shifts are measured and NTO usually predicts the same shift as Trad. Extensive analysis would be required, however, to be certain.

The most interesting of the tests is BH (Brillet and Hall), as this is the only one for which NTO and Trad differ with certainty with regard to results. BH used a Fabry-Perot interferometer that rotated with respect to the lab. A fraction of the light ray incident on the interferometer emerged directly from the far end. Another portion of the ray was reflected at the far end and forced to travel round trip, rear to front to rear, before emerging. The different portions interfered to form a fringe pattern. If the round trip speed of light were anisotropic, the time for it to travel back and forth inside the interferometer would vary with orientation of the apparatus. This, in turn, would cause the fringe pattern, and thus, the signal BH monitored, to vary. In Newtonian theory, this variation, peak-to-peak and to second order, is $\frac{1}{2}v^2/c^2$, where v is the maximum change from c of the speed of light. As shown by Klauber[47], the NTO effect on light transit time is quantitatively the same (though subtle calculational differences exist from the Newtonian analysis.)

The speed of the earth surface about its axis at the location of the BH test is .355 km/sec. For this, the amplitude of the variation via NTO theory should be

$$\frac{1}{4}\frac{v^2}{c^2} = \frac{1}{4}\left(\frac{.355}{3 \times 10^5}\right)^2 = 3.5 \times 10^{-13} \tag{1.24}$$

at twice the apparatus rotation rate. The BH test found a "persistent" $\sim 1.9 \times 10^{-13}$ signal at that rate and with fixed phase relative to the earth surface. They deemed this signal "spurious", because it seemed inexplicable. The character of the BH signal and its proximity in value to (1.24) should, of themselves, be intriguing. However, there is a secondary effect of light speed anisotropy on the BH signal.

The path of travel is altered slightly when the light ray direction is transverse to the principle direction of anisotropy. In a heuristic sense, the ray seems to be pushed "sideways". In the BH experiment, this would result in a shifting of the fringe pattern, and a concomitant change in the measured signal. Klauber[47] calculated this effect and found it dependent on certain dimensions of the apparatus, which are not known. However, by using values for these dimensions estimated from the figure of the apparatus shown in the BH report, he found an expected net signal from all effects of $\sim 2 \times 10^{-13}$.

3.4 Comparison to Selleri

As noted in Section 2.4.7, the Selleri theory, like the NTO approach, is based on what this author considers a physically defensible simultaneity scheme. The Selleri theory, on the other hand, predicts a null signal for the BH experiment.

It would be interesting to compare predictions of the Selleri theory to results of other tests such as Mössbauer.

3.5 GPS and Sagnac

I do not profess expertise in the GPS system, though I have noted earlier the remarks by Ashby, who does have extensive expertise. Those remarks appear consonant with NTO analysis and its requisite non-Einstein synchronization and local light speed anisotropy.

Furthermore, in the context of the thought experiment of Section 1.6.1, the traditional approach seems incapable of deriving the Sagnac effect from within the rotating frame. That is, considering the local physical speed of light to be isotropic does not seem sufficient to derive different arrival times for the cw and ccw light rays. This is not the case for the NTO approach, and in this context, the Sagnac experiment may be considered empirical support for it.

3.6 Future Experiments

Tobar[55][56] (WSMR in Table 1) expects to complete a modified version of the Michelson-Morley experiment, accurate to several orders of magnitude beyond that of BH, by the end of 2004. He will use a whispering spherical mode resonator and rotate it with respect to the lab. Preliminary analysis by the present author suggests that the WSMR experiment may be capable of detecting an NTO effect on light speed, if it exists, due to the surface speed of the earth.

3.7 Summary of Part 3

Only one non GPS/Sagnac experiment appears capable of distinguishing between the traditional and NTO approaches to relativistic rotation, that of Brillet and Hall. That test, sensitive to 10^{-15}, found a signal at $\sim 1.9 \times 10^{-13}$, which is strikingly close to the signal predicted by the NTO approach from the earth surface speed about its axis of rotation, and which is not predicted by the traditional approach.

Table 1. History of SRT Experiments

Symbol	Test Description	Authors (Year)	NTO Effect	Trad	NTO
MM	Original Michelson-Morley experiment	Michelson & Morley[57] (1887)	Accuracy too low. ~7-10 km/sec.	Y	Y
WW	Electric field effect of rotating magnetic insulator in magnetic field	Wilson and Wilson[58] (1913); Hertzberg et al [59] (2001)	NTO prediction = Trad[60]	Y	Y

Post MM	Repeats of MM interferometer tests	Kennedy (1926); Piccard & Stahel (1926-8); Michelson et al ((1929)	Null results: 1 km/sec to 7 km/sec accuracy	Y	Y
Joos	Version of MM	Joos[61] (1930)	Accuracy too low. ~1.5 km/sec.	Y	Y
KT	Original experiment on time dilation	Kennedy and Thorndike [62](1932)	Not rotated. Low accuracy ~10 km/sec	Y	Y
Ives-Stilwell	Doppler frequency time dilation in H canal rays	Ives and Stilwell [63](1938, 1941)	Accuracy 100X too low for NTO effect[64]	Y	Y
PartAcc	Particle accelerator time dilation on half lives	Mid 1900s to present	NTO prediction = Trad	Y	Y
ODM	Two opposite direction NH_3 maser beams. Ether wind Doppler variation as rotate.	Cedarholm et al [65] (1958)	Rotated 180^o, ~1.5 km/sec variation. Attributed to earth mag field.	?	Y
Hughes-Drever	Isotropy of nuclear energy levels. Doppler shift of photons emitted by 2 different atoms.	Hughes et al[66] and Drever (1960)	Significant analysis needed for NTO prediction.	Y	?
PDM	Perpendicular direction He-Ne masers. Rotated.	Jaseva, Townes et al[67] (1964)	Accuracy too low. Systematic signal as rotated.	?	Y
Mössbauer	Mössbauer rotor. Classical frequency shift different from SRT	Champeney et al[68] (1963); Turner and Hill [69] (1964)	NTO predicts same frequency change as Trad	Y	Y

HK	Time dilation on atomic clocks flown around world	Hafele and Keating[70] (1972)	NTO prediction = Trad	Y	Y
BH	Fringe shift in interferometer as rotate	Brillet and Hall [35] (1978)	2^{nd} order effect at 10^{-13}. NTO predicts[47]	N	Y
GPA	Gravity probe A. Maser on rocket and maser on ground. Classical Doppler varies from SRT.	Vessot and Levine [71] (1979), Vessot et al[72] (1980)	NTO shift = Trad. Analysis done in non-rotating earth centered frame.	Y	Y
Refract	Light split in air and glass. 1st order fringe effect in Galilean & Fresnel ether drag theories.	Byl et al[73] (1985)	1^{st} order effect NTO = Trad $\neq$ Galilean or Fresnel drag.[74]	Y	Y
NBS	Isotropy of nuclear energy levels. Doppler shift. Rotation of earth changed orientation.	Prestage[75] et al (1985) at National Bureau Standards	Apparatus not rotated. NTO effect = Trad	Y	Y
TPA	2 photon absorption in atomic beam. Doppler shift opposite directions affected by ether wind.	Kaivola et al[76] (1985); Riis et al [77] (1988)	Apparatus not rotated. Beam aligned N-S[78]. NTO effect = Trad	Y	Y
UWash	Isotropy of nuclear energy levels. Doppler shift. Rotation of earth changed orientation.	Lamoreaux et al[79] at Univ Washington (1986)	Apparatus not rotated. NTO effect = Trad	Y	Y
JPL	Jet Propulsion Lab. 2 earth fixed masers. Fiberoptic comparison.	Krisher et al[80] (1990)	Apparatus not rotated. NTO effect = Trad	Y	Y
LFV	Laser frequency variation as earth rotates: stabilized laser compared to stable cavity locked laser.	Hils and Hall[81] (1990)	Apparatus not rotated, plus accuracy too low for NTO effect.	Y	Y

Sat	GPS satellite test of SRT.	Wolf and Petit[82] (1997)	Analysis done in non-rotating earth centered frame. NTO effect = Trad.	Y	Y	
CORE	Cryogenic Optical Resonators measure anisotropy of light speed as earth rotates.	Braxmaier et al [83] (2002)	Apparatus not rotated. NTO effect = Trad.	Y	Y	
WSMR	Whispering spherical mode resonator Michelson-Morley experiment.	Tobar[55][56] (2004)	Appears capable of discerning between NTO and Trad	TBD	TBD	

Appendix. Deriving Sagnac Result from the Lab Frame

Consider Figure 1 of section 1.6.1 with time ($T > 0$) in the right side of the figure when the cw light pulse reaches the disk observer designated as T_1. Consider the time when the ccw pulse reaches the disk observer (not shown) as T_2. Then lengths traveled as seen from the lab by the ccw light pulse and the observer at T_1 must sum to equal the circumference, i.e.

$$cT_1 + \omega R T_1 = 2\pi R \qquad \rightarrow \qquad T_1 = \frac{2\pi R}{c + \omega R}. \tag{1.25}$$

Similarly, at time T_2

$$cT_2 = \omega R T_2 + 2\pi R \qquad \rightarrow \qquad T_2 = \frac{2\pi R}{c - \omega R}. \tag{1.26}$$

Hence, the arrival time difference in the lab is

$$\Delta T = T_2 - T_1 = \frac{2\omega R}{c^2} \frac{2\pi R}{(1 - \omega^2 R^2/c^2)} = \frac{4\omega A}{c^2(1 - \omega^2 R^2/c^2)}. \tag{1.27}$$

As is well known, the standard (physical) clocks on the disk rim run more slowly than the lab clocks by $\sqrt{1 - \omega^2 R^2/c^2}$, so the observer on the disk must measure an arrival time difference of

$$\Delta t_{phys} = \frac{4\omega A}{c^2\sqrt{1 - \omega^2 R^2/c^2}}. \tag{1.28}$$

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[7] See ref. [3], p. 223.

[8] Misner, C.W., Thorne, K.S., and Wheeler, J.A., *Gravitation* (Freeman, New York, 1973). See Chapter 6.

[9] Ref. [4], Stachel, p. 7.

[10] Transforming from the 4D Riemann flat space of the lab to the 4D space of the rotating frame implies the 4D space of the rotating frame is also Riemann flat. The traditional analysis does not contradict this, but claims the 2D surface of the rotating disk is curved within the flat 4D rotating frame space.

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[12] Some may contend that other meter sticks in the rotating frame (such as a meter stick on the far side of the rim) appear to have velocity with respect to a given rim observer. However, an observer fixed on the rim of the disk does *not* see the far side of the disk moving relative to her, since she, unlike the local Lorentz frame with the same instantaneous linear velocity, is rotating. There are two different local frames of interest here. Both have the same instantaneous rectilinear velocity as a point on the rim. But one rotates relative to the distant stars (at the same rate as the disk itself), and one does not. The latter is a local Lorentz frame. The former is not, and represents the true state of an observer anchored to the disk. The difference in kinematics between these two frames is significant, and is dealt with in Klauber, R., "Non-time-orthogonality, gravitational orbits and Thomas precession", gr-qc/0007018.

[13] Tartaglia, A., "Lengths on Rotating Platforms", *Founds. Phys. Lett.,* **12**(1.1), 17-28 (1999). Tartaglia and I have made the point in this paragraph independently.

[14] Nikolic', H., "Relativistic contraction and related effects in non-inertial frames", *Phys. Rev. A.* **61**, 032109 (2000), gr-qc/9904078; "Fermi co-ordinates and rotation", *Relativity on a Rotating Frame* in series *Fundamental Theories of Physics*, Editor Van der Merwe, A., Kluwer Academic Press (2003). Nikolic' presents a resolution of this based on the postulate that "although there is no relative motion among different points on a rotating disc, each point belongs to a different non-inertial frame". Though Nikolic' focuses on Fermi frames, for which this postulate may hold, the present author disagrees with it for the physical frame of the rotating disk. The present author considers the definition of the term "frame" to mean precisely that "there is no relative motion among different points therein."

[15] Selleri, F., "Noninvariant One-Way Speed of Light and Locally Equivalent Reference Frames", *Found. Phys. Lett.* **10**, 73-83 (1997). Selleri presents this argument quantitatively and makes a cogent argument for violation of the first postulate of SRT.

[16] For example, see Budden, T., "Geometric Simultaneity and the Continuity of Special Relativity", *Found. Phys. Lett.* ***11***(1.4), 343-357 (Aug 1998).

[17] Also, Sorge, F., "Local and Global Anisotropy in the Speed of Light", *Relativity on a Rotating Frame* in series *Fundamental Theories of Physics*, Editor van der Merwe, A., Kluwer Academic Press (2003).

[18] Also, Weber, T. "Some Elementary Relativistic Considerations of the Time and Geometry of Rotating Reference Frames", *Relativity on a Rotating Frame* in series *Fundamental Theories of Physics*, Editor van der Merwe, A., Kluwer Academic Press (2003).

[19] See, for example, Bergia, S., and Guidone, M., "Time on a Rotating Platform and the One-Way Speed of Light", *Found. Phys. Lett.,* **11**(1.6), 549-560 (Dec 1998).

[20] Also, refs. [11] and [18]

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[27] Ref. [19].

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[38] The texts and article listed below are among those that discuss physical vector and tensor components (the values one measures in experiment) and the relationship between them and coordinate components (the mathematical values that depend on the generalized coordinate system being used.) D. Savickas, "Relations between Newtonian Mechanics, general relativity, and quantum mechanics", *Am. J. Phys.*, 70(1.8), 798-806; I.S. Sokolnikoff, *Tensor Analysis*, Wiley & Sons (1951) pp. 8, 122-127, 205; G.E. Hay, *Vector and Tensor Analysis*, Dover (1953) pp 184-186; A. J. McConnell, *Application of Tensor Analysis*, Dover (1947) pp. 303-311; Carl E. Pearson, *Handbook of Applied Mathematics*, Van Nostrand Reinhold (1983 2nd ed.), pp. 214-216; Murry R. Spiegel, *Schaum's Outline of Vector Analysis*, Schaum, pg. 172; Robert C. Wrede, *Introduction to Vector and Tensor Analysis,* Dover (1972), pp. 234-235.

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[41] Physical components are introduced in ref. [8] on pg. 37, and used in many places throughout the text, though surprisingly, the relation between physical and coordinate components is not derived. It is used, however. See, for example, physical velocity components found in equation (31.5) on pg. 821.

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