Modeling of a split type air conditioner with integrated water heater

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Abstract

This paper presents a steady state simulation model to predict the performance of a small split type air conditioner with integrated water heater. The mathematical model consists of submodels of system components such as evaporator, condenser, compressor, capillary tube, receiver and water heater. These submodels were built based on fundamental principles of heat transfer, thermodynamics, fluid mechanics, empirical relationships and manufacturer’s data as necessary. The model was coded into a simulation program and used to predict system parameters of interest such as hot water temperature, condenser exit air temperature, evaporator exit air temperature, mass flow rate of refrigerant, heat rejection in the condenser and cooling capacity of the system. The simulation results were compared with experimental data obtained from an experimental rig built for validating the mathematical model. It was found that the experimental and simulation results are in good agreement.

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1. Introduction

In tropical countries such as Thailand, small split type air conditioners are generally used in residential and commercial buildings. In such establishments, electric water heaters are often used to generate hot water. Both air conditioner and electric water heater are generally the major energy consuming devices in the buildings. The number of air conditioners and electric water heaters has been increasing over the years, and this poses a serious problem to the country that largely depends on imported energy. Waste heat from air conditioners may be used to produce hot water. The benefits of doing this are two fold. One is elimination of the need to install an electric water heater, and the other is saving of electrical energy otherwise used in the electric water heater. These may be accomplished while the usefulness of the air conditioner for cooling is maintained.

At present, water heaters using waste heat from small split type air conditioners are commercially available in Thailand and are generally tailor made to the specific requirements of the users. Even though split type air conditioners with water heaters are successfully used, their performance and system design for application in Thailand have not been fully investigated, especially when both cooling and heating effects are desirable. Studies of heat pump hot water heaters operating in subtropical and cold countries have appeared in the literature. Some of such works include those of Ji et al. [1] and Baek et al. [2].

Heat pump and air conditioner models have been widely studied. The first heat pump computer model was developed by Hiller and Glicksman [3]. A number of models have been developed and used by researchers and manufacturers. There are also proprietary models that are not available for review in the literature. Some of the heat pump models available through the open literature are the MARKIII model developed at a national laboratory by Fischer et al. [4], the HPSIM model developed at NBS by Domanski and Didion [5] and the heat pump grain drying models by Theerakulpisut [6]. All of these models are for air-to-air heat pumps. Room air conditioning system modeling (RACMOD) developed by Mullen [7] was also available. This model was based on the governing equations of
### Nomenclature

- **A**: total area (m²)
- **C**: capacity rate (kW/K), capacity rate ratio = $C_{\text{min}}/C_{\text{max}}$ (dimensionless)
- **Cₚ**: specific heat (kJ/kg/K)
- **COP**: coefficient of performance (dimensionless)
- **DP**: total pressure drop (kPa)
- **DPH**: high side pressure drop (kPa)
- **DPL**: low side pressure drop (kPa)
- **DSH**: degree of superheat (°C)
- **f**: fraction (dimensionless)
- **g**: Gravitational acceleration (m/s²)
- **Grₚ**: Grashof Number (dimensionless)
- **h**: enthalpy (kJ/kg), heat transfer coefficient (kW/m²/K)
- **hₐₚ**: enthalpy of saturated air evaluated at refrigerant temperature (kJ/kg)
- **hₐₚₘ**: enthalpy of saturated air evaluated at mean water film temperature (kJ/kg)
- **k**: thermal conductivity (kW/m/K)
- **Le**: Lewis Number (dimensionless)
- **m**: mass flow rate (kg/s)
- **mₗ**: mass of water in hot water tank (kg)
- **N**: number of transfer units (dimensionless)
- **Nₜₚ**: Nusselt Number (dimensionless)
- **P**: refrigerant pressure (kPa), power (kW)
- **Pₐ**: compressor discharge pressure (MPa)
- **Pₐₙ**: compressor suction pressure (MPa)
- **Pr**: Prandtl Number (dimensionless)
- **Pₜₚ**: pressure drop (kPa)
- **q**: heat transfer rate (kW)
- **qₜₚ**: heat rejection rate of heating coil calculated from enthalpy difference between inlet and exit of two phase region (kW)
- **qₜₚₘ**: heat rejection rate of heating coil in two phase surface evaluated from overall heat transfer coefficient equation (kW)
- **Ra**: Rayleigh number (dimensionless)
- **ρ**: density (kg/m³)
- **t**: temperature (°C)
- **tₐₚ**: temperature at beginning of interval of one minute (°C)
- **tₚ**: overall heat transfer coefficient (kW/m²/K)
- **t₉₉ₚ**: overall heat transfer coefficient for wet surface based on enthalpy difference (kg/m²/s)
- **w**: air humidity ratio (kg water/kg dry air)

### Greek letters

- **β**: thermal expansion coefficient (K⁻¹)
- **ϕ**: heat transfer coefficient (dimensionless)
- **ε**: effectiveness of heat exchangers (dimensionless)
- **ηₑ**: electric motor efficiency of rotary compressor (dimensionless)
- **ηₘ**: mechanical efficiency of rotary compressor (dimensionless)
- **μ₉**: dynamic viscosity of saturated refrigerant (Pa.s)
- **ν**: water viscosity (m²/s)
- **π₈**: eighth pi term (dimensionless)

### Subscripts

- **a**: air, acceleration
- **as**: air side
- **c**: condenser, condensing
- **cap**: capillary tube
- **comp**: compressor
- **d**: dehumidification, dew point, dry
- **dsh**: desuperheating section
- **e**: evaporator, evaporating
- **f**: saturated liquid, friction
- **fin**: fin
- **g**: saturated vapour
- **h**: heating coil
- **i**: inlet, inside
- **ll**: liquid line
- **m**: mean
- **max**: maximum
- **min**: minimum
- **o**: outlet, outside
- **r**: refrigerant
- **rb**: return bend
- **rc**: receiver
- **rs**: refrigerant side
- **sc**: subcooled section
- **sh**: superheated section
- **st**: straight tube
- **tp**: two phase section
- **w**: wet, water
- **ws**: water side

Some of these subscripts are also used in combination. For example, atpo refers to air at outlet of two phase section and scsta to subcooled liquid, straight tube and component acceleration.
the ORNL heat pump model developed by Fisher and Rice [8] and modified by O’Neal and Penson [9]. All the models of heat pumps and air conditioners were used to study air-to-air heat transfer phenomena. None of the aforementioned models attempted to study the case where waste heat is to be recovered from the air conditioning system. It is of great interest to understand the influence of a water heater addition to the system that performs a dual task of cooling and heating.

The main objective of this study is to build a reliable mathematical model of an air conditioner with integrated hot water heater for the purpose of system study. It is expected that the model will be sufficiently accurate for studying the system performance when it is designed to perform an extra task of heating water in addition to its normal cooling task. Parametric study of the system may also be performed to understand the principal parameters influencing the system performance. The model will be validated by experimentation to ensure that it may be used for the intended purpose.

This model was developed from Theerakulpisut’s model [6]. The difference between this model and Theerakulpisut’s model lies in the fact that Theerakulpisut’s model contains reciprocating compressor and thermostatic expansion valve submodels, whereas this study proposes to model a system that uses a rotary compressor and a capillary tube. As a result of the inclusion of a hot water tank between the compressor and the condenser, the computer program of this study is also more complicated since the refrigerant state at the condenser inlet may be superheated, two phase or subcooled. Modeling of the system is next outlined.

2. Mathematical model

A schematic of the air conditioning system with integrated water heater proposed in this study is shown in Fig. 1, and its operating cycle is shown in Fig. 2. The system consists of the major components of the air conditioner, namely compressor, evaporator, condenser, capillary tube and receiver and an additional water heater that also serves as a water storage tank. It should be noted that during normal operation, hot water is expected to be withdrawn from the tank, and the tank is supplied with water. At this stage of the study, it is assumed that water is not withdrawn from the tank. However, the model will be further extended to cover the case where hot water is withdrawn from the tank. The mathematical model of each component can be described as follows.

2.1. Capillary tube and compressor model

The capillary tube model adopts the equation from ASHRAE [10]. In this method, the Buckingham pi theorem was applied to the physical factors and fluid properties that affect capillary tube flow. The result of this analysis was a group of eight dimensionless pi terms. The process in the capillary tube is assumed to be adiabatic. The entering state of refrigerant can be subcooled or a mixture of liquid and vapour. The effect of coiling the capillary tube is taken into account. This coiling of the capillary tube approximately reduces the refrigerant mass flow rate by 5% when compared with that in a straight tube [11]. Therefore, the eighth dimensionless pi terms equation is multiplied by 0.95 as shown in Eq. (1). The procedure for determining \( \pi_8 \) may be found in ASHRAE [10].

\[
\dot{m}_{\text{cap}} = 0.95\pi_8 d_{\text{cap}} \mu_f \tag{1}
\]

The compressor model is obtained by curve fitting the manufacturer’s data [12] to give Eq. (2) for the refrigerant mass flow rate and Eq. (3) for the compressor power input.
The work input to the refrigerant during the compression process \(w_{a5}\) can be calculated from Eq. (4). Efficiencies of rotary compressors were given by Ozu and Itami [13]. Typical mechanical efficiency \((\eta_m)\) and electric motor efficiency \((\eta_t)\) are both recommended to be 0.85.

\[
w_{a5} = \eta_m \eta_t P_{comp} m_t
\]  

Then, the refrigerant enthalpy at the compressor exit \((h_s)\) can be calculated from

\[
h_s = h_4 + w_{a5}
\]

where the refrigerant enthalpy at the compressor inlet \((h_4)\) is determined from the receiver model.

### 2.2. Condenser model

The condenser model consists of a large number of equations. Since some of these are quite elaborate, they will not all be presented here and only the main equations will be described. Heat transfer in the condenser was modeled using the NTU-\(\varepsilon\) method. The condenser heat transfer area is divided into three zones, namely desuperheating, two phase and subcooled zones. After leaving the water heater, the refrigerant entering the condenser can be superheated, two phase or subcooled. Modeling strategy of the condenser is as follows.

#### 2.2.1. Desuperheating zone

If the condenser inlet refrigerant state is superheated, part of the condenser area is in the desuperheating zone. The Newton–Raphson method is used to solve Eq. (6) for the number of transfer unit \((N_{dsh})\), and the desuperheating fraction \((f_{dsh})\) is calculated from Eq. (7). The overall heat transfer coefficient in the desuperheating zone \((U_{dsh})\) is computed from Eq. (8), while the fin efficiency \((\phi)\) is evaluated by the equation by Charters and Theerakulpisut [14]. The refrigerant side heat transfer coefficient \((h_{ras})\) is computed from the well known Dittus–Boelter equation, and the air side heat transfer coefficient \((h_{ads})\) equation is calculated from the equation of Webb [15].

\[
m_{t,comp} = \frac{1}{3600} \left[ 130.12950 - 1.31674 t_c + 0.00696 t_c^2 
+ (9.90989 - 0.23284 t_c - 0.00194 t_c^2) t_e 
+ (0.43826 - 0.01524 t_c + 0.00015 t_c^2) t_e^2 \right] \tag{2}
\]

\[
P_{comp} = \frac{1}{1000} \left[ 389.81950 + 9.84761 t_c + 0.06207 t_c^2 
+ (-18.09224 + 0.63628 t_c - 0.00332 t_c^2) t_e 
+ (1.37331 - 0.06466 t_c + 0.00068 t_c^2) t_e^2 \right] \tag{3}
\]

Eq. (1) is used to compute the refrigerant mass flow rate to compare with the value obtained from Eq. (2) until agreement within a specified tolerance is achieved.

The heat transfer rate of the desuperheating section \((q_{dsh})\) and the air temperature at the desuperheating zone exit, which is equal to the air temperature at the two phase zone inlet \((t_{atpi})\) are calculated from Eqs. (9) and (10), respectively

\[
q_{dsh} = C_r(t_s - t_c) \tag{9}
\]

\[
t_{atpi} = t_{aci} + q_{dsh}/C_a \tag{10}
\]

#### 2.2.2. Two phase zone

The computation method is the same as that in the desuperheating zone, but the refrigerant side heat transfer coefficient in the two phase region is evaluated from the equation proposed by Traviss et al. [16]. The procedure for calculating the two phase fraction is as follows:

\[
e_{tp} = 1 - e^{-N_{tp}} \tag{11}
\]

\[
e_{tp} = \frac{C_a(t_{atpo} - t_{atpi})}{C_{min}(t_c - t_{atpi})} \tag{12}
\]

\[
f_{tp} = A_{tp}/A_c \tag{13}
\]

\[
N_{tp} = \frac{U_{tp} A_{tp}}{C_{min}} = \frac{U_{tp} A_{tp}}{C_a} \tag{14}
\]

By combining Eqs. (11), (12) and (14),

\[
f_{tp} = \frac{C_a}{U_{tp} A_c} \ln \left( \frac{t_c - t_{atpi}}{t_c - t_{atpo}} \right) \tag{15}
\]

Calculation of \(f_{tp}\) by Eq. (15) requires \(t_{atpo}\), which can be calculated from

\[
t_{atpo} = t_{atpi} + m_r h_{ig}/C_a \tag{16}
\]

Note that Eq. (16) is valid only if the refrigerant enters the two phase section as saturated vapour and leaves as saturated liquid. The heat transfer rate for the two phase section is readily calculated from

\[
q_{tp} = m_r h_{ig} \tag{17}
\]

The subcooled zone may exist in the condenser, and the subcooled fraction \((f_{sc})\) can now be calculated from

\[
f_{sc} = 1 - f_{tp} + f_{dsh}, \quad f_{tp} + f_{dsh} \leq 1 \tag{18}
\]

or \(f_{sc} = 0 \quad f_{tp} + f_{dsh} > 1 \tag{19}\)

If the subcooled zone does not exist, i.e. \(f_{sc} = 0\), the two phase fraction must be determined from

\[
f_{tp} = 1 - f_{dsh} \tag{20}
\]

and the heat transfer rate from

\[
q_{tp} = e_{tp} C_{min}(t_c - t_{atpi}) \tag{21}
\]
The air temperature at the two phase exit \( (t_{\text{atpo}}) \) is, therefore, calculated from
\[
    t_{\text{atpo}} = t_{\text{api}} + q_{\text{tp}}/C_a \tag{22}
\]

2.2.3. Subcooled zone

If the subcooled fraction is greater than zero, heat transfer for the subcooled region in the condenser is evaluated from the following equations. The calculation procedure is similar to that of the desuperheating section.

\[
    N_{\text{sc}} = \frac{U_{\text{sc}}f_{\text{sc}}A_e}{C_{\text{min}}} \tag{23}
\]

\[
    \varepsilon_{\text{sc}} = 1 - \exp \left\{ N_{\text{sc}}^0.22 \frac{\exp \left\{ -CN_{\text{sc}}^0.78 \right\} - 1}{C} \right\} \tag{24}
\]

\[
    t_0 = t_c - \frac{\varepsilon_{\text{sc}}C_{\text{min}}(t_c - t_{\text{atpo}})}{C_{\text{sc}}} \tag{25}
\]

Note that the overall heat transfer coefficient in the subcooled zone \( (U_{\text{sc}}) \) takes the same as Eq. (8) used for the evaluated \( U_{\text{dsh}} \). The heat transfer rate for the subcooled zone may now be calculated from
\[
    q_{\text{sc}} = C_{\text{sc}}(t_c - t_0) \tag{26}
\]
and the air temperature at the condenser outlet from
\[
    t_{\text{aco}} = t_{\text{atpo}} + q_{\text{sc}}/C_a \tag{27}
\]
The total heat transfer rate in the condenser is simply the sum of the heat transfer rates of the three zones.
\[
    q_e = q_{\text{dsh}} + q_{\text{tp}} + q_{\text{sc}} \tag{28}
\]

2.3. Evaporator model

Heat transfer area in the evaporator is divided into two zones, which are the superheated and two phase zones.

2.3.1. Superheated zone

Heat transfer in the superheated zone of the evaporator may be calculated from the same equations as were used in the desuperheating zone of the condenser.

2.3.2. Two phase zone

It will be assumed that dehumidification may take place only on the two phase surface. This assumption can be checked by comparing the calculated air dew point and the calculated mean fin temperature at the interface between the two phase and the superheated zones. In fact, the two phase surface may be entirely dry, partially wet or totally wet, depending on the incoming air condition.

2.3.2.1. Entirely dry two phase surface. If the air temperature passing through the evaporator is higher than its dew point, it is not dehumidifying. Therefore, the heat transfer is evaluated in the entirely dry two phase surface condition as Eq. (29)
\[
    q_{\text{tp}} = \dot{m}_x(h_{\text{ge}} - h_2) \tag{29}
\]

The refrigerant enthalpy at the exit of the two phase section is the enthalpy at saturation corresponding to the pressure at the exit of the evaporator \( (h_{\text{ge}}) \), whereas the inlet refrigerant enthalpy \( (h_2) \) must be calculated by initially assuming the refrigerant quality at the inlet \( (x_2) \). This assumed \( x_2 \) will be later checked as the calculation proceeds. Once the heat transfer is calculated by Eq. (29), the air temperature at the exit of the two phase section \( (t_{\text{atpo}}) \) may be calculated from
\[
    t_{\text{atpo}} = t_{\text{aci}} - q_{\text{tp}}/C_a \tag{30}
\]

The two phase fraction is computed from Eq. (31) and the overall heat transfer coefficient in the dry two phase region \( (U_{\text{dip}}) \) is calculated in a similar fashion as in the case of the overall heat transfer coefficient in the desuperheating region of the condenser. The refrigerant side two phase heat transfer coefficient can be computed from the equation proposed by Chaddock and Noerager [17] and Sthapak et al. [18].
\[
    f_{\text{tp}} = \frac{C_a}{U_{\text{dip}}A_e} \ln \left[ \frac{t_{\text{aci}} - t_c}{t_{\text{atpo}} - t_c} \right] \tag{31}
\]

It must be remembered that the calculation of \( (f_{\text{tp}}) \) according to Eq. (31) is possible because of the initial assumption of the inlet refrigerant quality \( (x_2) \), which leads to the determination of \( t_{\text{atpo}} \), which may not necessarily be correct. The procedure for checking this follows. Once \( f_{\text{tp}} \) is calculated by Eq. (31), the number of transfer units of the two phase region \( (N_{\text{tp}}) \) is calculated from Eq. (32).
\[
    N_{\text{tp}} = \frac{U_{\text{dip}}A_{\text{tp}}}{C_{\text{min}}} = \frac{U_{\text{dip}}f_{\text{tp}}A_e}{C_a} \tag{32}
\]
and the effectiveness of the two phase section from
\[
    \varepsilon_{\text{tp}} = 1 - e^{-N_{\text{tp}}} \tag{33}
\]

Using the definition of heat exchanger effectiveness, one may write
\[
    \varepsilon_{\text{tp}} = \frac{C_a(t_{\text{aci}} - t_{\text{atpo}})}{C_{\text{min}}(t_{\text{aci}} - t_c)} = \frac{(t_{\text{aci}} - t_{\text{atpo}})}{(t_{\text{aci}} - t_c)} \tag{34}
\]

By equating Eq. (33) and (34), Eq. (35) is obtained for calculating the air temperature at the two phase zone exit of the evaporator \( (t_{\text{atpo}}) \).
\[
    t_{\text{atpo}} = t_{\text{aci}} - (t_{\text{aci}} - t_c)(1 - e^{-N_{\text{tp}}}) \tag{35}
\]

Eq. (35) may now be used to calculate \( t_{\text{atpo}} \) using the \( N_{\text{tp}} \) previously calculated from Eq. (32). This new \( t_{\text{atpo}} \) is then used in Eq. (30) to calculate \( q_{\text{tp}} \). The new value of \( q_{\text{tp}} \) is then substituted into Eq. (29) to calculate \( h_2 \), which is used to calculate the new \( x_2 \). The new \( x_2 \) is then compared with the initially assumed value. If they do not agree within a specified tolerance, the new \( x_2 \) is used to begin the next loop of iteration until the desired agreement is achieved.

Once the actual operating cycle is established, the evaporator exit air temperature may be obtained from
\[
    t_{\text{aco}} = t_{\text{atpo}} - q_{\text{sh}}/C_a \tag{36}
\]
where \( q_{\text{sh}} \) is

\[
q_{\text{sh}} = C_{\ell}(\text{DSH})
\]  

(37)

The degree of superheat (DSH) was experimentally obtained for the system in this investigation. The values of DSH varied from 1.66 to 1.97 °C, therefore the average DSH of 1.8 °C is used in the model.

The total heat transfer in the evaporator for the entirely dry two phase surface condition is evaluated from Eq. (38).

\[
q_e = q_{\text{tp}} + q_{\text{sh}}
\]  

(38)

So far, the evaporator has been modeled on the assumption of an entirely dry evaporator surface. However, this is rarely the case under normal operating conditions. It is, therefore, necessary to determine whether there is dehumidification of water vapour on the evaporator coil. The criteria for establishing this will be outlined next.

2.3.2.2. Two phase surface with dehumidification.

In the evaporator, dehumidification begins at the location where the mean fin temperature is equal to the dew point of the incoming air (\( t_e \)). Thus, the air temperature at the location where dehumidification just begins (\( t_{\text{ad}} \)) is computed from Eq. (39).

\[
t_{\text{ad}} = t_a - \phi(1 - U_{\text{ad}}A_e/h_wA_{\text{sh}})t_e
= 1 - \phi(1 - U_{\text{ad}}A_e/h_wA_{\text{sh}})
\]  

(39)

Eq. (39) may be used to calculate \( t_{\text{ad}} \) which is then compared with the air temperature at the exit of the two phase section (\( t_{\text{atp0}} \)) calculated from Eq. (35). If \( t_{\text{atp0}} \) is greater than \( t_{\text{ad}} \), the whole two phase surface is dry, and the modeling approach is as presented in Section 2.3.2.1. If \( t_{\text{atp0}} \) is less than \( t_{\text{ad}} \), the two phase section is partially or entirely wet. It can be further established whether the surface is entirely wet by comparing \( t_{\text{ad}} \) with the air temperature at the evaporator inlet (\( t_{\text{ai}} \)). If \( t_{\text{ai}} \) is less than \( t_{\text{ad}} \), the whole surface is wet, otherwise it is partially wet.

The Newton–Raphson method is used to solve Eq. (40) for the number of transfer units of the superheated region (\( N_{\text{sh}} \)) assuming a value of \( t_{\text{atp0}} \) in Eq. (40), which must be checked later for agreement with the new value calculated from the analysis of the two-phase section. Then the superheated fraction (\( f_{\text{sh}} \)) and the two phase fraction (\( f_{\text{tp}} \)) are computed from Eqs. (41) and (42), respectively.

\[
\frac{C_{\ell}(t_3 - t_e)}{C_{\min}(t_{\text{atp0}} - t_e)} = 1 - \exp\left\{\frac{N_{\text{sh}}^{0.22}}{C_{\text{sh}}} \exp\left(-CA_{\text{sh}}^{0.78}\right) - 1\right\}
\]  

(40)

\[
f_{\text{sh}} = \frac{N_{\text{sh}}}{U_{\text{sh}}A_e}
\]  

(41)

\[
f_{\text{tp}} = 1 - f_{\text{sh}}
\]  

(42)

If the two phase surface is determined by the previously discussed criterion to be partially wet, further division of the surface into dry and wet two phase sections is necessary. From the definition of heat exchanger effectiveness and the effectiveness equation similar to Eq. (33), the dry two phase fraction takes the following form:

\[
f_{\text{dtp}} = \frac{C_{\ell} U_{\text{dtp}}A_e}{\ln \left[\frac{t_{\text{at}} - t_e}{t_{\text{ad}} - t_e}\right]}
\]  

(43)

The wet two phase surface fraction (\( f_{\text{wtp}} \)) is then given by

\[
f_{\text{wtp}} = 1 - f_{\text{sh}} - f_{\text{dtp}}
\]  

(44)

It is important to note that the dry surface fraction (\( f_{\text{dtp}} \)) is zero when the two phase fraction is entirely wet and that Eq. (44) still holds.

The heat transfer rate from the air to the refrigerant in the dry two phase section may be calculated from

\[
q_{\text{dtp}} = C_{\ell}(t_{\text{at}} - t_{\text{ad}})
\]  

(45)

The method for analyzing the wet evaporator in this study is adopted from Threlkeld [19]. The method was used by Theerakulpisut and Priprem [20], and it was found that it accurately predicted the performance of wet evaporator surfaces. The method is quite complicated and involves a number of equations. For the purpose of outlining the calculation procedure, only the principal equations of the methods are presented as follows.

The humidity ratio change across the wet surface may be calculated from

\[
\frac{dw}{dh_a} = \left( \frac{h_a - \bar{h}_{\text{aswm}}}{w - \bar{w}_{\text{aswm}}} \right) + \left( h_w - 2501Le \right)^{-1}
\]  

(46)

where

\[
h_w = 2501 + 1.805w
\]  

(47)

The complexity of Eq. (46) lies in the method for calculating the mean value of the mean water film temperature for evaluating \( \bar{h}_{\text{aswm}} \) and \( \bar{w}_{\text{aswm}} \). The procedure for establishing this was outlined by Theerakulpisut and Priprem [20].

Consideration of Eq. (46) reveals that an analytical solution is difficult and numerical integration is deemed appropriate. By substituting all the terms pertaining to the inlet condition of the wet section of the evaporator, \( \left( \frac{dw}{dh_a} \right)_i \), is evaluated. As an approximation, one may write

\[
\Delta w = w_{ei} - w_{\text{atp0}} = \frac{dw}{dh_a} \left( \frac{dw}{dh_a} \right)_i
\]  

(48)

which gives

\[
w_{\text{atp0}} = w_{ei} - (h_{\text{ad}} - h_{\text{atp0}}) \left( \frac{dw}{dh_a} \right)_i
\]  

(49)

where

\[
h_{\text{ad}} = t_{\text{ad}} + w_{ei}(2501 + 1.805t_{\text{ad}})
\]  

(50)

It can be shown that the enthalpy of the air at the outlet section of the two phase section is given by

\[
h_{\text{atp0}} = h_{\text{ad}} + (h_{\text{ad}} - h_{\text{ad}})e^{-U_{\text{as}}f_{\text{atp0}}t_{\text{a}}/m_s}
\]  

(51)

Now, it is possible to predict the air temperature at the outlet of the two phase section

\[
t_{\text{atp0}} = (h_{\text{atp0}} - 2501w_{\text{atp0}})/(1 + 1.805w_{\text{atp0}})
\]  

(52)
The heat transfer rate in the two phase section may now be calculated from
\[ \dot{q}_{tp} = \dot{m}_t (h_{aei} - h_{atpo}) \]  (53)
Hence, the total heat transfer rate of the evaporator is given by
\[ \dot{q}_e = \dot{q}_{tp} + \dot{q}_{sh} \]  (54)
The refrigerant enthalpy at the inlet of the evaporator \( h_2 \) can be calculated from
\[ h_2 = h_3 - \frac{q_e}{\dot{m}_t} \]  (55)
The new refrigerant quality \( x_2 \) is calculated from Eq. (56) and compared with the assumed value
\[ x_2 = \frac{(h_2 - h_{le})}{(h_{ge} - h_{le})} \]  (56)
If the new \( x_2 \) does not agree with the assumed value, the new value is used in the next round of iteration. This procedure is repeated until \( x_2 \) converges to a specified tolerance.

2.4. Liquid, suction and discharge lines

Heat transfer in the liquid, suction and discharge lines can be evaluated in the form of free convection. For example, heat loss from the refrigerant in the liquid line to ambient air is calculated from
\[ \dot{d}q_{ll} = \frac{U_{ll}}{\pi d A_{ll}} \frac{dA_{ll}}{\dot{m}_t C_{pr}} dt \]  (57)
which can be integrated to give the refrigerant temperature at the outlet of the liquid line as
\[ t_1 = t_{am} + (t_6 - t_{am}) \exp \left(-\frac{U_{ll} A_{ll}}{\dot{m}_t C_{pr}} \right) \]  (58)

It should be mentioned that the connecting pipe between the hot water tank and the condenser is assumed to be insulated. Therefore, the heat transfer in this connecting pipe is negligible. The refrigerant temperature at the heating coil exit \( (t_f) \) is equal to the refrigerant temperature at the condenser inlet \( (t_8) \).

2.5. Receiver model

The receiver model is assumed to be under a steady state condition, and there is no insulation. The receiver may be considered as a container of vapour receiving heat from the environment. In this condition, sensible heat will be added to the vapour refrigerant by natural convection from the ambient air. Thus,
\[ \dot{m}_t C_{pr} (t_f - t_4) = h_{rc} A_{rc} (t_{rm} - t_{am}) \]  (59)
where the refrigerant temperature at the receiver inlet \( (t_f) \) is evaluated from the suction line model.

2.6. Pressure drop in the system

Pressure drop in the system consists of pressure drops in the major components of the system and connecting pipes.

2.6.1. Pressure drops in the evaporator, the condenser and the heating coil

There are six components of the pressure drop in the evaporator, which are evaluated from Eq. (60).
\[ \Delta P_e = \left( \Delta P_{tpstf} + \Delta P_{tpsta} \right) + \Delta P_{tpbrf} + \left( \Delta P_{shstf} + \Delta P_{shsta} \right) + \Delta P_{shrbf} \]  (60)
The straight tube two phase pressure drop can be calculated by using the relationship proposed by Traviss et al. [16] and the return bend two phase pressure drop by the relationship given by Geary [21]. While the straight tube single phase pressure drop can be computed from the well known Darcy-Weisbach equation [10], the return bend single phase pressure drop makes use of the relationship proposed by Ito [22].

Pressure drop in the condenser is similarly calculated by Eq. (61). It should be noted that there are additional pressure drop components because the equation must also cover pressure drops in the subcooled zone. This equation is also used to compute pressure drops in the heating coil in the water tank.
\[ \Delta P_c = \left( \Delta P_{tpstf} + \Delta P_{tpsta} \right) + \Delta P_{tpbrf} + \left( \Delta P_{shstf} + \Delta P_{shsta} \right) + \Delta P_{shrbf} \]  (61)

2.6.2. Pressure drops in the connecting pipes

Single phase pressure drops in the connecting pipes are evaluated by the Darcy-Weisbach equation. When the connecting pipes contain two phase fluid, the pressure drop is calculated in a similar manner as in the case of the two phase pressure drop in the evaporator.

2.7. Refrigerant properties

The refrigerant in the air conditioner in this study is R-22. Thermodynamic properties of the refrigerant are evaluated by the equations presented by Cleland [23]. The physical properties are calculated from equations obtained by curve fitting the data of ASHRAE [10] and the equations presented by Stoecker and Jones [24].

2.8. Water heater model

The water tank, which contains a heating coil, is located between the condenser and the compressor. Heat transfer on the refrigerant side is divided into three zones as shown in Fig. 3.
The refrigerant state at the heating coil inlet, state 6, is determined from the discharge line model. All of the heating coil is first assumed to be a desuperheating surface. The desuperheating section of the heating coil area \( A_{hdsh} \) is equal to the total heating coil area \( A_h \). The refrigerant temperature at the heating coil exit \( (t_f) \) is calculated from
\[ t_f = t_{wm} + (t_6 - t_{am}) \exp \left(-\frac{U_{hdsh} A_{hdsh}}{\dot{m}_t C_{pr}} \right) \]  (62)
and the desuperheating area of the heating coil \( A_{\text{hdsh}} \) can be calculated from
\[
A_{\text{hdsh}} = \frac{-\dot{m}_c C_{\text{pr}}}{U_{\text{hdsh}}} \ln \left( \frac{t_c - t_{\text{wm}}}{t_c - t_{\text{wm}}} \right)
\]  \( (71) \)

Then, it is assumed that all of the remaining area is a two phase surface
\[
A_{\text{hp}} = A_h - A_{\text{hdsh}}
\]  \( (72) \)

The exit refrigerant quality of the heating coil \( \chi_f \) is assumed in order to calculate the overall heat transfer coefficient in the two phase region \( U_{\text{hp}} \). Then, the heat rejection rate in the two phase section of the heating coil, which is evaluated from Eq. (73), is compared with the heat rejection rate calculated from the enthalpy difference between the inlet and the exit of the two phase region, Eq. (74)
\[
q_{\text{hp}} = U_{\text{hp}} A_{\text{hp}} (t_c - t_{\text{wm}}) \quad (73)
\]
\[
q_{\text{hp}} = \dot{m}_c (h_{gc} - h_f) \quad (74)
\]

where the refrigerant enthalpy at the heating coil exit \( h_f \) can be evaluated from
\[
h_f = h_{ic} + \chi_f h_{ig} \quad (75)
\]

If the values of \( q_{\text{hp}} \) and \( q_{\text{hp}} \) do not agree within an acceptable tolerance, \( \chi_f \) is varied, and \( U_{\text{hp}}, q_{\text{hp}} \) and \( q_{\text{hp}} \) are recalculated until the desired agreement between \( q_{\text{hp}} \) and \( q_{\text{hp}} \) is achieved. During the procedure to compare \( q_{\text{hp}} \) and \( q_{\text{hp}} \), if \( h_f \) is less than the enthalpy of saturated liquid refrigerant at the condenser pressure \( (h_{ic}), \) the subcooled zone may exist. In this case, the heat transfer rate of the two phase section of the heating coil \( q_{\text{hp}} \) can be calculated from
\[
q_{\text{hp}} = \dot{m}_c h_{ic} \quad (76)
\]

Then, the heat transfer area of the two phase section of the heating coil \( A_{\text{hp}} \) is evaluated from
\[
A_{\text{hp}} = \frac{q_{\text{hp}}}{U_{\text{hp}} (t_c - t_{\text{wm}})} \quad (77)
\]

The subcooled area of the heating coil \( A_{\text{suc}} \) may now be calculated from
\[
a_{\text{suc}} = A_h - A_{\text{hdsh}} - A_{\text{hp}} \quad (78)
\]

The refrigerant temperature at the exit of the heating coil at \( t_j \) is calculated from
\[
t_j = t_{\text{wm}} + (t_c - t_{\text{wm}}) \exp \left( -U_{\text{suc}} A_{\text{suc}} / \dot{m}_c C_{\text{pr}} \right) \quad (79)
\]

Therefore, the heat transfer rate of the subcooled section of the heating coil \( q_{\text{suc}} \) can be calculated from
\[
q_{\text{suc}} = \dot{m}_c (h_{ic} - h_{f}) \quad (80)
\]

During the calculational procedure to establish the agreement between \( q_{\text{hp}} \) and \( q_{\text{hp}} \), if \( h_f \) is greater than \( h_{ic}, \) the subcooled section does not exist. Therefore, \( q_{\text{suc}} = 0, t_j = t_c \) and \( q_{\text{hp}} \) is evaluated from
The total heat transfer rate in the heating coil is simply the sum of the heat transfer rates of the three zones

\[ q_{htp} = \dot{m}_r \left( h_{gc} - h_t \right) \]  \hfill (81)

The refrigerant mass flow rate \( q_h \) is

\[ q_h = q_{hsh} + q_{htp} + q_{hsc} \]  \hfill (82)

It should be noted that the mean water temperature \( t_{wm} \) is first assumed at the beginning of the calculation procedure, the new value of mean water temperature \( t_{wm,new} \) will be calculated from Eq. (84). If the \( t_{wm,new} \) does not agree with the assumed value, the new value is used in the next round of iteration. This procedure is repeated until \( t_{wm} \) converges to a specified tolerance.

\[ t_{we} = t_{ws} + \frac{q_p x 60}{m_w C_{pw}} \]  \hfill (83)

\[ t_{wm,new} = \frac{t_{ws} + t_{we}}{2} \]  \hfill (84)

3. Model simulation program

The mathematical model is coded into a computer program using FORTRAN 90. This simulation program can be used to calculate parameters of prime interest such as water temperature in the hot water tank, condenser exit air temperature, evaporator exit air temperature, refrigerant mass flow rate, heat rejection in the condenser and in the hot water tank and cooling capacity of the system. All dimensions of the system components such as fin and tube geometry of the evaporator and the condenser, dimensions of the capillary tube, and dimensions of the connecting pipes are included in the program. The input data of the program are the initial water temperature and the degrees of superheat of the refrigerant at the evaporator exit.

To begin simulation, the program reads the first set of operating condition data, i.e. the air dry bulb and wet bulb temperatures at the evaporator inlet and the air dry bulb temperature at the condenser inlet. The evaporator exit pressure \( P_3 \), compressor discharge pressure \( P_6 \), high side pressure drop (DPH) and low side pressure drop (DPL) are first assumed. The capillary tube inlet refrigerant temperature \( t_1 \) is initially assumed to be less than the saturated refrigerant temperature corresponding to the condenser pressure \( t_c \), and therefore, the refrigerant state is assumed to be subcooled at the capillary tube inlet. Then, the refrigerant mass flow rate \( \dot{m}_{r,cap} \) is computed by the capillary tube model and compared with the refrigerant mass flow rate \( \dot{m}_{r,comp} \), which is calculated from the rotary compressor model. If they do not agree within an acceptable tolerance, \( P_3 \) is changed to a new value and the whole calculation procedure is repeated until the calculated evaporator inlet air temperature agrees with the actual value. Then, the refrigerant state at the capillary tube inlet must be checked. If \( h_1 \) is greater than \( h_{fc} \), the refrigerant state at the capillary tube inlet is two phase. Therefore, the refrigerant quality at the capillary tube inlet \( x_1 \) is computed, and the program will proceed to then compare with the assumed value. If they do not agree within a specified tolerance, the \( x_{1,new} \) will be used to repeat the calculation process until convergence is achieved.

It should be noted that in the first loop of analysis, the pressure drops across all components of the system are assumed to be negligible so that \( P_1 \) may be taken as \( P_5 \), and \( P_2 \) as \( P_3 \). The effects of pressure drops may now be taken into account to improve the analysis as illustrated in Fig. 4.

At this point, simulation of the model has been finished for the first set of operating condition data. The program then reads the second set and the operating condition data from the input data file and proceeds in a similar manner until the end of the simulation.

4. Model validation

An experimental rig of an air conditioning system with integrated water heater was built for the purpose of verifying the model. Basically, the experimental rig consists of six major components of the system, namely compressor, receiver, evaporator, condenser, capillary tube and a water heater. The schematic of the experimental set up is shown in Fig. 5. The hot water tank is located between the compressor and the condenser. Note that in this arrangement, water in the hot water tank is not withdrawn while being heated. This configuration represents the situation where hot water is produced and stored for later use. It also represents the simplest and least expensive arrangement if waste heat from the air conditioner is to be used for heating.
Fig. 4. Flow chart of the simulation program of a split type air conditioner with integrated water heater.
water. The cooling capacity of the air conditioner is 12,000 Btu/h (3.517 kW), while the hot water tank capacity is 102 l. The heating coil in the hot water tank consists of two similar coils as shown in Fig. 6. The coil is made from copper tubing with 1.27 mm diameter. Each heating coil consists of six straight tubes each with a length of 0.59 m., two straight tubes that are 0.635 m. in length and 7 U-bends with 25 mm radius with a total length of 5.3 m. The two coils were connected in series, thus having the total length of 10.6 m. The hot water tank is insulated with 50.8 mm synthetic rubber insulation.

5. Measuring equipment

The experimental rig was equipped with measuring equipment to determine the refrigerant and air states, power inputs, refrigerant and air mass flow rates and hot water temperature.

5.1. Temperature measurement

As indicated in Fig. 5, temperature measurements are made by K-type thermocouples at sixteen locations, three locations \((t_4, t_5, t_9)\) for refrigerant temperatures, four locations \((t_{14} - t_{17})\) for air dry bulb and air wet bulb air temperatures at the inlet and the exit of the evaporator and one location \((t_{18})\) for air dry bulb air temperature at the condenser inlet, four locations \((t_{19} - t_{22})\) for air dry bulb temperatures at the condenser exit and four locations \((t_{10} - t_{13})\) for the water temperature in the hot water tank. All the thermocouples were calibrated with a GRANT water bath with an accuracy of ±0.5 °C (5–90 °C) and connected to the CAMBELL model CX-23 data logger interfaced with a desk top computer.

Note that the condenser exit air temperature was measured at four different positions, and therefore, the condenser exit temperature was taken as the average value of these four positions. Similarly, the water temperature in the hot water tank was also taken as the average value of four readings at four different positions within the tank.

It should also be noted that in measuring the air wet bulb temperatures, the thermocouples were covered with wicks that were fed with distilled water from bottles through plastic tubes leading to the wicks. The feed rate of water was experimentally determined to avoid excessive addition of water to the air system. Wet bulb readings were verified by a brand new Testo model 625 having an accuracy ±0.5 °C (−10 to 60 °C). Since these thermocouples were placed in the air stream through the evaporator with sufficient air velocity, the wet bulb readings were quite accurate provided that the wicks were sufficiently wet.

Refrigerant temperatures were also measured by K-type thermocouples. The thermocouples were installed on the outside surface of the copper tubes and thermal paste was applied to ensure good contact between the thermocouples and the tube surface.

5.2. Refrigerant pressure measurement

Refrigerant pressures were measured at five locations \((P_3, P_4, P_5, P_7, P_9)\) by Bourdon pressure gauges. These pressure gauges were new and factory calibrated with an accuracy of ±1% of full scale.
5.3. Refrigerant mass flow rate

The refrigerant mass flow rate was measured by a HED-LINE orifice type model HLIT-205-2G flow meter with an accuracy of ±2% of full scale. This flow meter is intended for single liquid phase measurement, so it was installed in the liquid line. A sight glass was installed before the flow meter to ensure that the refrigerant was in liquid phase when readings were taken.

5.4. Air flow rate

Air flow rates through the condenser and the evaporator were measured by measuring air velocities at the exits of the condenser and the evaporator. The air velocities were measured by a Testo model 425 anemometer with telescopc probe. The anemometer has an accuracy of ±0.05 m/s (0–20 m/s). Before taking measurements, the fan speed of the evaporator was set at the highest speed. The air velocities for the condenser and the evaporator were taken at ten different locations. The average values of these ten locations were used in calculation of the air mass flow rates through the condenser and the evaporator. Since the experimental rig was run with fixed condenser and evaporator fan speeds, the air mass flow rates were determined only once for every experimental run.

5.5. Power input measurement

The power input to the system was measured by a FLUKE 39 power meter with an accuracy of ±2% (0–10 kW). The meter was new and factory calibrated.

6. Experimental procedure

Before each experimental run, the data logger and the computer were turned on to check that all the measuring instruments were ready for the experiment. Initial operating condition, as defined by the air dry bulb and wet bulb temperatures at the evaporator inlet, the air dry bulb temperature at the condenser inlet and the initial water temperature in the water tank, were recorded. Once the experimental rig was turned on, thermocouple readings were recorded in a data file at intervals of one minute. The recorded data included the operating conditions, which were used for the model simulation, as well as other parameters for model validation. During the course of a preliminary run of the experiment, it was found that refrigerant pressures, refrigerant mass flow rate and power inputs were almost constant. These parameters were, therefore, recorded at an interval of ten minutes. Since the experiment was mainly for the purpose of model validation, a period of three hours for each experimental run was judged to be sufficient.

7. Results and discussion

For the purpose of validating the mathematical model, ten experimental runs to cover a wide range of operating conditions were conducted, and important parameters were recorded for comparison with the simulation results. These parameters include compressor exit refrigerant temperature, evaporator exit air temperature, condenser exit air temperature, compressor discharge and suction pressures, refrigerant mass flow rate and compressor power input. The actual operating conditions of the experiments, as determined by the inlet conditions of the air flow through the evaporator, the inlet air temperature through the condenser and the initial water temperature in the hot water tank, were used to simulate the model. The simulation results of all the ten runs were found to be in good agreement with the experimental results. Thus, only the simulation and experimental results of a typical experimental run will be presented.

Typical experimental and simulation results are presented in Figs. 7–15. Fig. 7 compared predicted and experimental temperatures of refrigerant at the compressor exit. Except for the first ten minutes of the experiment, this figure reveals that the model overpredicted the experimental data by 10%. Figs. 8 and 9 show plots of experimental and predicted values of the evaporator exit air temperature and the condenser exit air temperature. The percentage deviations of the experimental values from the predicted values were 4% for the evaporator exit air temperature and 3% for the condenser exit air temperature.

As indicated in Fig. 10, the experimental compressor discharge and suction pressures deviated from the predicted values within ±5%. The experimental and predicted refrigerant mass flow rates are shown in Fig. 11, and the experimental and predicted compressor power inputs are shown in Fig. 12. Fig. 11 reveals that the model overpredicted the experimental data by 6%, whereas Fig. 12 indicates that the model underpredicted the experimental value by 7%.

Fig. 13 compares experimental and predicted water temperatures in the hot water tank. This figure reveals that the model overpredicted the experimental data by approximately 7% during the first hour of the experiment and that agreement between the experimental and predicted values were very good after that.

Figs. 14 and 15 present experimental data and predicted values for heat rejection rates at the condenser and cooling capacity of the evaporator, respectively. The percentage deviations from the predicted value were ±24% for the condenser heat rejection rate and ±21% for the cooling capacity of the evaporator. It should be noted that the condenser heat rejection rate, and the cooling capacity are derived quantities. Equipment errors in the measurement of air temperature and humidity can contribute to significant errors of the quantities. An error analysis was conducted to determine the greatest possible errors from the calculation of these two quantities. The method of analysis as
Fig. 7. Predicted and experimental compressor exit refrigerant temperatures.

Fig. 8. Predicted and experimental evaporator exit air temperatures.

Fig. 9. Predicted and experimental condenser exit air temperatures.
Fig. 10. Predicted and experimental compressor discharge and suction pressures.

Fig. 11. Predicted and experimental refrigerant mass flow rates.

Fig. 12. Predicted and experimental compressor power inputs.
Fig. 13. Predicted and experimental water temperatures in the hot water tank.

Fig. 14. Predicted and experimental heat rejection rates at the condenser.

Fig. 15. Predicted and experimental cooling capacities of the evaporator.
described in Ref. [26] was based on a careful specification of the uncertainties in the various primary experimental measurements. It was found that the greatest possible errors were ±24.37% for the condenser heat rejection rate and ±24.58% for the evaporator cooling rate. These errors were, therefore, comparable with the actual discrepancies between the experimental and simulation values.

8. Conclusions

A mathematical model of a split-type air conditioner with integrated water heater has been developed by detailed modeling of the system components. The model is based on the fundamental principles of thermodynamics, heat transfer and fluid mechanics. Empirical equations are used to describe the heat and mass transfer and pressure losses occurring in these components. The principle advantages of the model based on this approach are that the model provides flexibility for design changes and that the model can be expected to reflect the real behavior of the system, as evidenced from the experimental and predicted results in this study. The model was validated against experimental data, and it was found that the model is quite accurate in predicting important system parameters. The model will be further used to study the performance of other system configurations under different operating conditions. Parametric study of different systems may also be performed to understand the principal parameters influencing the system performance.

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