Rapid Performance Evaluation and Optimal Sizing of Dry Cyclone Separators

Alexia A. Economopoulou¹ and Alexander P. Economopoulos²

Abstract: Cyclones are generally less efficient than other kinds of equipment, but their simple construction, low energy requirements, and ability to operate at high temperatures and pressures make them attractive for cleaning up gases. Despite the simplicity in construction and operation, complex mathematical formulations are used for predicting the collection efficiency of particles of a given diameter. These must be numerically integrated, along with the inlet particle-size-distribution functions that are appropriate in each application, in order to obtain the overall cyclone efficiency. In this paper, the above cumbersome procedure is simplified through nomographs allowing rapid, yet rigorous, estimation of the overall cyclone efficiencies based on two alternative and well-established approaches and on the sole assumption of a lognormal particle-size distribution. Along with the above, pressure drop and limiting inlet velocity correlations are also considered, and each of the above nomographs is combined with others, providing direct graphical representation of the so far obscure relationships among cyclone diameter, overall efficiency, and gas pressure drop or flow rate. The paper thus affords an overview of cyclone behavior over a wide range of conditions, offering direct solutions to both cyclone performance and optimal cyclone design problems.

DOI: 10.1061/(ASCE)0733-9372(2002)128:3(275)

CE Database keywords: Performance evaluation; Cyclone separators; Emissions; Air pollution control; Dust.

Introduction

Cyclones are relatively simple mechanical collectors operating by the principle of imparting centrifugal force to the particles to be removed from the carrier gas. They are relatively inexpensive to install and operate, easy to maintain, and able to function under high temperatures, pressures, and dust loadings. Large cyclones achieve good efficiencies for particles with diameters upward of 10 μm, while small, high-efficiency ones can be effective in collecting particles down to the 5-μm range. Because of the above advantages, cyclones are used widely in industry for removing particles from processes and for dust control.

Cyclones are made in a variety of configurations, the most common of which is shown schematically in Fig. 1. Particulate matter is being separated from a carrier gas by transforming the velocity of an inlet stream into a double vortex confined within the cyclone. In this the entering gas spirals downward at the outside and upward at the inside of the cyclone outlet. The particles, because of their inertia, tend to move toward the outside wall, from which they precipitate and are led to a receiver. This design is called a vertical reverse-flow cyclone with tangential inlet and it is the principal subject of the analysis that follows.

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Note. Associate Editor: Mark J. Rood. Discussion open until August 1, 2002. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on June 19, 2000, approved on July 24, 2001. This paper is part of the Journal of Environmental Engineering, Vol. 128, No. 3, March 1, 2002. ©ASCE, ISSN 0733-9372/2002/3-275–285/$8.00+$0.50 per page.

A cyclone size is characterized by the principal diameter $D$, while its shape or configuration is determined by seven geometric ratios $a/D$, $b/D$, $D_e/D$, $S/D$, $h/D$, $H/D$, and $B/D$. Several sets of these ratios have evolved from experience so as to result in efficient designs, and six of the most widely used ones are listed in Table 1. The above geometrical parameters, along with the particle density and size distribution and key operating variables, such as gas flow rate, temperature, and pressure, are used as inputs to cyclone simulation models.

Several models, each of which comprises a fairly complex set of equations, have been used successfully in predicting the collection efficiency (as a function of particle diameter) of cyclones. The overall cyclone efficiency can be computed numerically, combining the predicted efficiencies for each particle size from the above models with inlet particle-size-distribution data from the emission source under consideration. Other important cyclone performance parameters are the pressure drop, which is directly related to the electric energy requirements that contribute significantly to the operating costs, and the salutation velocity, which defines the gas flow rate beyond which the efficiency starts declining due to excessive re-entrainment of the centrifugally separated solid particles.

The computational requirements in cyclone performance simulations make it necessary to use computers. Even more complex are cyclone system design problems, where for the required efficiency the gas pressure drop has to be balanced against the number of parallel cyclones to be used. The solution of these problems normally involves a cumbersome trial and error procedure, in the context of which alternative designs are tested and evaluated. The objective here is to provide direct graphical solutions to both cyclone simulation and design problems. In addition to the above use, a further objective is to enable the compilation of credible size-specific particle emission inventories (Economopoulou and Economopoulos 2001, 2002).
In either case, to further facilitate the use of nomographs, relations for the required physical properties of common gases are provided in this paper while a compendium of lognormal particle-size-distribution parameters is given elsewhere (Economopoulou and Economopoulos 2001).

### Selected Cyclone Performance Models

In cyclones, the collecting forces operate in a manner that depends upon particle size, shape, and density. Consequently, different particle sizes are collected with different degrees of effectiveness. Thus, for a given cyclone under specific operating conditions, a grade efficiency can be defined for particles with size $d_i$.

In our analysis, two alternative modeling approaches will be applied for the evaluation of grade efficiencies of solid particles, those developed by Dietz (1981) and by Leith and Licht (1972). The latter appears satisfactory even when conditions of high temperatures, high pressures, and high velocities of inlet gas dominate the chamber. The former is suitable over a fairly extended range of operating temperatures and for moderate velocities and dust loadings ($c<10$ g/m$^3$) (Licht 1988).

#### Dietz Model for Grade Efficiencies

The model of Dietz (1981) involves a detailed analysis of the internal volume and particle residence time within the cyclone. For this purpose, three regions are considered: an inlet region, a downflow region, and an upflow region. Within each region the assumption is made that the radial particle transport is dominated by turbulent mixing. Based on these considerations, Dietz expressed the grade efficiency of particles belonging to the $i$th size class, $E_{f_i}$, with a relation that after some elementary transformations and conversion to metric units has the following form:

$$E_{f_i} = 1 - \left[ K_0 - (K_1^2 + K_2)^{0.5} \right] \exp \left( -\frac{\pi (2K_5 - K_4)}{K_6 K_7} \frac{Stk}{K_a K_b} \right)$$

where

### Table 1. Cyclone Geometric Ratios and “Standard” Design Configurations

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
<th>“High Efficiency”</th>
<th>“General Purpose”</th>
<th>“Experimental”</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D/D$</td>
<td>Body diameter</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$K_a = a/D$</td>
<td>Inlet height</td>
<td>0.5</td>
<td>0.44</td>
<td>0.5</td>
</tr>
<tr>
<td>$K_b = b/D$</td>
<td>Inlet width</td>
<td>0.2</td>
<td>0.21</td>
<td>0.25</td>
</tr>
<tr>
<td>$K_s = S/D$</td>
<td>Outlet length</td>
<td>0.5</td>
<td>0.5</td>
<td>0.625</td>
</tr>
<tr>
<td>$K_v = D_v/D$</td>
<td>Gas-outlet diameter</td>
<td>0.5</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$b/D$</td>
<td>Cylinder height</td>
<td>1.5</td>
<td>1.4</td>
<td>2.0</td>
</tr>
<tr>
<td>$H/D$</td>
<td>Overall height</td>
<td>4.0</td>
<td>3.9</td>
<td>4.0</td>
</tr>
<tr>
<td>$B/D$</td>
<td>Dust-outlet diameter</td>
<td>0.375</td>
<td>0.4</td>
<td>0.25</td>
</tr>
<tr>
<td>$K_l = l/D$</td>
<td>Natural length</td>
<td>2.48</td>
<td>2.04</td>
<td>2.30</td>
</tr>
<tr>
<td>$G$</td>
<td>Eq. (9)</td>
<td>551.2</td>
<td>698.7</td>
<td>402.9</td>
</tr>
<tr>
<td>$W_1$</td>
<td>Eq. (43)</td>
<td>0.0262</td>
<td>0.0424</td>
<td>0.0201</td>
</tr>
<tr>
<td>$W_2$</td>
<td>Eq. (39)</td>
<td>54.759</td>
<td>54.327</td>
<td>95.430</td>
</tr>
</tbody>
</table>
The pressure drop in cyclones can be predicted satisfactorily from the following form:

\[ K_0 = \frac{1}{2} \left[ 1 + K_r^{2\gamma} \left( 1 + \frac{K_r K_h}{2 \pi K_{\text{Stk}}} \right) \right] \]  

\[ K_1 = \frac{1}{2} \left[ 1 - K_r^{2\gamma} \left( 1 + \frac{K_r K_h}{2 \pi K_{\text{Stk}}} \right) \right] \]  

\[ K_2 = (K_r)^{2\gamma} \]  

\[ \gamma = 1 - (1 - 0.67D_0^{0.14}) \left( \frac{T}{283} \right)^{0.3} \]  

\[ \text{Stk} = \frac{\rho_p Q d_i^2}{18 \times 10^{12} \mu D_{ab}^2} \]  

**Leith and Licht Model for Grade Efficiencies**

The theoretical approach of Leith and Licht (1972) to calculating cyclone collection efficiencies is based on the concept of continual radial backmixing of the uncollected particles. The relevant cyclone collection efficiencies is based on the concept of convection, conduction, and diffusion processes. The theoretical approach of Leith and Licht is developed for the Dietz and Licht inlet velocity relations.

The gas pressure drop in cyclones is known to decrease as the dust loading increases. Masin and Koch (1986) produced the following quantitative expression for this effect:

\[ u_i = \frac{Q}{ab} \]  

\[ N_H = K(ab/D_e^2) \]  

\[ N_H = 11.3(ab/D_e^2)^2 + 3.33 \]  

\[ \Delta P = \Delta P_0 [1 - 0.0086e^{0.5}] \]  

where \( \Delta P_0 \) is the uncorrected cyclone’s pressure drop (for \( c = 0 \)) as computed from Eq. (16) and \( \Delta P \) is the corrected one for dust loading \( c \) (g/m³).

**Graphical Analysis of Cyclone Design and Performance**

The objective here is to develop a set of nomographs solving simultaneously the cyclone efficiency, pressure drop, and limiting inlet velocity relations.

The graphical solution of the overall cyclone efficiency problem is developed for the Dietz (1981) as well as for the Leith and Licht (1972) formulations. The former is the first to appear in the literature. A graphical solution for the latter was first developed by Leith (1979), but the present one is streamlined with the analysis in the section “Selected Cyclone Performance Models” and the requirements for size-specific inventories as detailed elsewhere (Economopoulos and Economopoulos 2001). In order to facilitate the use of nomographs, relations are also provided for the convenient estimation of required gas properties.
Calculation of Overall Efficiency from Grade Efficiencies

Given the size distribution of particles in terms of a cumulative fraction less than size $d_i$, $M_0 = M_0(d_i)$ ($0 < M_0 < 1$) and the grade efficiency of the collector by an equation such as $E_{f_i} = E_{f}(d_i)$, then the overall efficiency in terms of mass can be calculated by integrating the grade efficiency over the particle size distribution:

$$E_f = \int_0^1 E_{f_i} \, dM_0$$

The overall penetration can then be expressed as:

$$p = 1 - E_f = \int_0^1 (1 - E_{f_i}) \, dM_0$$

A simplifying and commonly used assumption is that the inlet dust particles follow a lognormal distribution. If $d_m$ and $\sigma_g$ are the mass median diameter and standard deviation, Eq. (23) may be written as follows (e.g., Licht 1988):

$$p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} \ln \left( \frac{1 + K_{2g}^2}{2} + \Gamma \right) \! dt$$

where the variate $t$ is defined as:

$$t = \frac{\ln d_i - \ln d_m}{\ln \sigma_g}$$

It should be noted that the complex relations of any selected grade efficiency model, such as those of Dietz (1981) and of Leith and Licht (1972), constitute an integral part of Eq. (24). As a result, Eq. (24) cannot be solved analytically and numerical methods have to be employed for estimating the overall penetration.

Nomographs for Overall Efficiency based on Dietz Formulation

Introducing Eqs. (1)–(4) and (6) into Eq. (24) and rearranging, we obtain the following expression:

$$p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left\{ -\frac{t^2}{2} - \ln \left( \frac{1 + K_{2g}^2}{2} + \Gamma \right) \right\} \! dt$$

where:

$$\Gamma = \frac{9}{\pi K_f} \left( \frac{\rho_p Q}{10^{12} K_{2g}^2 \mu D^2} \right) \sigma_g^2$$

and

$$\Delta = K_{2g}^2 \left( \frac{2 K_S - K_a}{18 K_S K_a} \right)^2 \left( \frac{\rho_p Q}{10^{12} K_{2g}^2 \mu D^2} \right) \sigma_g^2$$

The overall penetration can then be expressed as:

$$p = 1 - E_f = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left\{ -\frac{t^2}{2} - \ln \left( \frac{1 + K_{2g}^2}{2} + \Gamma \right) \right\} \! dt$$

The upper nomographs in Figs. 2–6 provide the sought graphical solution of Eq. (33) for each of the “standard” design configurations of Table 1. To derive these nomographs, the overall efficiency $E_f$ is estimated from the numerical solution of Eqs. (25)–(31) for a series of $\ln \sigma_g$ and $C_m$ values, the latter generated through valid combinations of $Q$, $D$, and $d_m$ values. For each $\ln \sigma_g$ and $C_m$ combination, the numerical calculation of the integral in Eq. (26) involved more than 300 particle diameter increments, which were more densely spaced in the low-diameter region. The nomographs generated allow direct assessment of the overall efficiency according to the Dietz (1981) theoretical approach. The “general-purpose” configurations of “Lapple” and “Swift” are accommodated in a single nomograph as they have similar geometric ratios and, for all practical purposes, result in identical solutions.

Nomograph for Overall Efficiency based on Leith and Licht Formulation

Introducing the grade efficiency $E_f = f(D_m, \ln \sigma_g)$, Eq. (32) can be solved graphically to provide direct assessment of the overall collection efficiency $E_f$ as a function of the parameters $D_m$ and $\ln \sigma_g$. The nomograph in Fig. 7, derived through numerical solution of Eq. (34), provides the graphical solution sought.

Graphical Relation of Maximum Cyclone Diameter and Overall Efficiency

The inlet gas to salutation velocity ratio needs to be kept within bounds for proper cyclone operation. As discussed above, the maximum cyclone efficiency is attainable when
Fig. 2. Nomographs for simulating and designing cyclones with “High-efficiency Stairmand” configuration based on Dietz formulation.

Fig. 3. Nomographs for simulating and designing cyclones with “High-efficiency Swift” configuration based on Dietz formulation.
Fig. 4. Nomographs for simulating and designing cyclones with “General purpose Lapple or Swift” configuration based on Dietz formulation.

Fig. 5. Nomographs for simulating and designing cyclones with “Experimental Dirgo and Leith” configuration based on Dietz formulation.
Introducing Eqs. (17) and (21) and the geometric ratios $K_e = D_e/D$, $K_a = a/D$, and $K_b = b/D$ into Eq. (37) and solving the resulting equation for $Q$, we obtain the maximum design flow rate

$$Q_{\text{max}} = W_2 \left( \frac{\mu p_p}{\rho_g} \right) D^{2.2}$$

where

$$W_2 = \frac{3022.1 K_a K_b^{2.2}}{1 - K_b}$$

The values of the parameter $W_2$ for all “standard” design configurations are listed in Table 1 and for any other cyclone configuration can be computed from Eq. (39).

Under maximum cyclone efficiency operation conditions, Eq. (38) can be introduced into Eq. (31) to eliminate $Q$. Thus $Cd_m$, one of the two parameters involved in Eq. (33), is given by the following equation:

$$Cd_m = \frac{W_2^{0.5} d_m \rho_p}{\rho_g} \frac{1}{K_e D^{0.2}}$$

For any “standard” cyclone design configuration the parameters $W_2$ and $K_e$ in the above equation are constant with their values listed in Table 1. The parameter $\gamma$, defined by Eq. (5), depends on $D$ and $T$. To simplify the problem, we note that calculation of the parameter $\gamma$ for a constant value of $T (T = 100^\circ C)$, rather than the actual one in each case, introduces negligible error in the estimation of $Cd_m$ over a wide temperature range. Under such conditions, the parameter $K_e/2$ depends only on $D$ and Eq. (40) can be written in the following functional form:

$$D_{\text{max}} = f \left( Cd_m, \frac{d_m \rho_p}{\rho_g} \right)$$

The middle nomograph in Figs. 2–6 provides a graphical solution of Eq. (41), yielding the maximum diameter a cyclone can have to attain the specified value of Ef (and $Cd_m$) with the inlet gas velocity within proper bounds.
Graphical Relation among Overall Efficiency, Diameter, and Pressure Drop

Introducing Eqs. (17) and (19) into Eq. (16) we obtain the following expression for the pressure drop:

\[ \Delta P = W_1 \rho_g \left( \frac{Q}{D^2} \right)^2 \]  (42)

where

\[ W_1 = \left( \frac{11.3}{K_e} + \frac{3.33}{K_e^2 K_b} \right) \left( 2 g \rho_w \right)^{1/2} \]  (43)

For any given “standard” design configuration the parameter \( W_1 \) above is constant and its values are listed in Table 1. For any other cyclone configuration \( W_1 \) can be easily estimated from Eq. (43) using \( \rho_w = 1.000 \text{ kg/m}^3 \) and \( g = 9.81 \text{ m/s}^2 \).

Solving Eq. (42) for \( Q \) we obtain

\[ Q = D^2 \sqrt{\frac{\Delta P}{W_1 \rho_g}} \]  (44)

Introducing Eq. (44) into Eq. (31) to eliminate \( Q \), we obtain

\[ C_{d_m} = \frac{1}{10^9 W_1^{0.25} \rho_p^{0.5} \mu^{0.25}} \Delta P^{0.25} \left( \frac{1}{K_e^2 D^{0.25}} \right) \]  (45)

Calculation of \( K_e^2 \) with a fixed value of \( T (T = 100^\circ \text{C}) \), as before, makes it possible to write Eq. (45) in the following functional form:

\[ D = f(C_{d_m}, L \cdot \Delta P) \]  (46)

where

\[ L = \frac{d_m^2 \rho_p^2}{10^{20} \rho_g \mu^2} = \frac{(d_m/10)^4 (\rho_p/1000)^2}{\rho_p (10^3 \mu)^2} \]  (47)

The lowest nomographs in Figs. 2–6 provide a graphical solution of Eq. (46). Combined use of the top and bottom nomographs in each figure offers a direct graphical representation of the relationships between overall efficiency, diameter, and gas pressure drop.

Gas Density and Viscosity Data

The gas density \( \rho_g \) can be computed from the ideal gas law, Eq. (48), with the gas density at standard conditions \( (\rho_g)_{\text{std}} \) having the values of 1.293 and 1.297 for ambient air and for flue gas, respectively.

\[ \rho_g = (\rho_g)_{\text{std}} \frac{273.16}{T} P \]  (48)

The gas viscosities are calculated as a function of temperature from the relation of Neufeld et al. (1972), and the Lennard-Jones potential coefficients tabulated by Reid et al. (1977). The results for ambient air and flue gas with a typical molar composition of \( \text{O}_2 = 3.64\% \), \( \text{N}_2 = 75.2\% \), \( \text{CO}_2 = 11.32\% \), \( \text{H}_2\text{O} = 9.83\% \), and \( \text{SO}_2 = 0.01\% \) are plotted in the diagram of Fig. 8 over the range of 0 to 500°C. The diagram of Fig. 8 is applicable over all pressures normally employed in cyclone operations, as the gas viscosity is unaffected by pressure at this low reduced-pressure range (Reid et al. 1977).

Use of Nomographs

The nomographs in Figs. 2–7 enable solution of both cyclone performance and design problems. In either case, data are needed about the operating conditions (\( Q_{\text{tot}}, T, \) and \( P \)), particle properties \( (\rho_p, d_m, \) and \( \sigma_g \)), and cyclone configuration.

Typical particle property values for many sources of interest are given elsewhere (Economopoulou and Economopoulois 2001), while the gas density and viscosity \( (\rho_g, \) and \( \mu \)), which are also required, can be obtained from Eq. (48) and Fig. 8 as functions of \( T \) and \( P \).

Solution of Existing Cyclone Performance Problems

In this type of problem, designers are assumed to know the number of parallel cyclones, \( n \), their configuration and diameter \( D \) and seek to estimate \( \Delta P \) and \( \text{Ef} \). For this, they can proceed as follows:

1. Check whether \( Q \leq Q_{\text{max}} \) so as to ensure that the inlet gas velocity remains within proper limits, where \( Q = Q_{\text{tot}}/n \) and \( Q_{\text{max}} \) is calculated from Eq. (38) with \( W_2 \) obtained from Table 1 or computed from Eq. (39).
2. Calculate \( \Delta P \) from Eq. (42) with \( W_1 \) obtained from Table 1 or computed from Eq. (43).
3. For \( \text{Ef} \) based on the Dietz formulation, compute \( C_{d_m} \) from Eqs. (31) and (5), select the appropriate set of nomographs from Figs. 2–6 according to the cyclone configuration, and read \( \text{Ef} \) from its top nomograph as a function of \( C_{d_m} \) and \( \ln \sigma_g \).
4. For \( \text{Ef} \) based on the Leith and Licht formulation, compute the parameters \( A_d \) and \( N \ln \sigma_g \) from Eqs. (5), (8), and (36), with \( G \) obtained from Table 1 or computed from Eq. (9), and read \( \text{Ef} \) from Fig. 7.

Solution of New Cyclone Design Problems

In this type of problem designers are assumed to know \( \text{Ef} \) and seek to balance \( \Delta P \) against the required number of cyclones, \( n \), so as to minimize the annualized capital investment and operating costs. For this they can proceed as follows:

1. Select the appropriate set of nomographs from Figs. 2–6 on the basis of the cyclone configuration and read the parameter \( C_{d_m} \) from the top nomograph as a function of the specified \( \text{Ef} \) and \( \ln \sigma_g \).
2. Calculate the parameters \( \ln(d_{m,p}/p_g) \) and \( L \), the latter from Eq. (47).

3. From the middle nomograph, read \( D_{\text{max}} \), the maximum diameter the cyclone can have for delivering the specified efficiency, as a function of the above calculated \( \ln(d_{m,p}/p_g) \) and \( C_d\). If \( D_{\text{max}} \) is small, outside the lower range of curves, the specified efficiency is too high for the defined particle-size distribution and density and cannot be attained by the cyclone configuration selected. If, on the other hand, \( D_{\text{max}} \) is more than 3 m (outside the upper range of curves), there is no practical cyclone diameter limitation.

4. Select any desirable value of \( D \) subject to \( D < D_{\text{max}} \). From the bottom nomograph read the value of \((L \cdot \Delta P)\) as a function of \( D \) and \( C_d \) and calculate \( \Delta P \) from \( \Delta P = (L \cdot \Delta P)/L \). Alternatively, select any desirable value of \( \Delta P \), calculate the parameter \( L \cdot \Delta P \), read the value of \( D \) from the bottom nomograph, and check whether \( D < D_{\text{max}} \). The above combination of \( D \) and \( \Delta P \) delivers the specified \( \text{EF} \) with inlet gas velocity within proper bounds. Indeed, as a cyclone with \( D = D_{\text{max}} \) delivers \( \text{EF} \) with \((u_i/u_s) = 1.25 \), a cyclone with a smaller diameter can deliver higher efficiency with \((u_i/u_s) = 1.25 \) or the same efficiency with \((u_i/u_s) < 1.25 \).

5. Calculate the gas flow rate handled by each cyclone, \( Q \), from Eq. (44) with \( W_1 \) obtained from Table 1, and the required number of parallel cyclones from the relation \( n = Q_{\text{tot}}/Q \). The calculated \( n \) is rounded to the higher integer if somewhat lower efficiency and pressure drop than the specified ones are preferred, or to the lower integer if a somewhat higher efficiency and pressure drop than the specified ones are preferred.

6. If necessary, repeat steps 4 and 5 above with different starting values of \( D \) or \( \Delta P \) until \( \Delta P \) and \( n \) are properly balanced. Selection of smaller diameters results in lower pressure drops, and hence smaller operating costs, but in greater numbers of cyclones, and hence higher capital investment costs. The objective here is to find the optimal design that minimizes the sum of amortized capital investment and operating costs.

### Example

A cyclone system with the “High-efficiency Swift” configuration is to serve as a precleaner in a dry bottom pulverized coal boiler. The design is to be based on typical particle-size distribution and the following design conditions:

- Particle density: \( \rho_p = 2,300 \text{ kg/m}^3 \)
- Maximum flow rate: \( Q_{\text{tot}} = 5 \text{ actual m}^3/\text{s} \)
- Gas temperature: \( T = 160^\circ \text{C} \)
- Gas pressure: atmospheric

We are asked to design a system capable of delivering an overall efficiency of 94.5%.

### Solution

The particles emitted from dry bottom pulverized coal boilers can be assumed to have a lognormal size distribution with typical parameters \( d_{\text{m},p} = 33.45 \mu\text{m} \) and \( \sigma_g = 5.42 \mu\text{m} \) (Economopoulos and Economopoulos 2001). For \( T = 160^\circ \text{C} \), Eq. (48) and Fig. 8 yield \( \rho_p = 0.8179 \text{ kg/m}^3 \) and \( \mu = 2.21 \times 10^{-5} \text{ kg/m} \cdot \text{s} \). We can now apply the step-by-step design procedure described above as follows:

- **Step 1**: For the “High-efficiency Swift” configuration we use Fig. 3. From its top nomograph and for the specified \( \text{EF} = 0.945 \) and \( \ln \sigma_g = 1.69 \) we read \( C_d = 1.14 \).
- **Step 2**: We compute \( \ln(d_{m,p}/p_g) = 11.45 \), and from Eq. (47) \( L = 165.8 \).
- **Step 3**: From the middle nomograph we read \( D_{\text{max}} = 1.3 \text{ m} \).
- **Step 4**: We can now use the bottom nomograph and experiment with various \( D \) or \( \Delta P \) values, until we reach a satisfactory system design. For the same \( C_d = 1.14 \) (therefore \( \text{EF} = 0.945 \)) and \( D = 0.6 \text{ m} \) we read \((L \cdot \Delta P)/L = 0.181 \).
- **Step 5**: From Eq. (44) we compute \( Q = 0.822 \text{ actual m}^3/\text{s} \). Thus, the system requires \( n = Q_{\text{tot}}/Q = 6 \) parallel cyclones.
- **Step 6**: In order to reduce further the above computed pressure drop, we can repeat steps 4 and 5 trying a smaller diameter. For \( D = 0.4 \text{ m} \) and \( C_d = 1.14 \) we read \((L \cdot \Delta P)/L = 16.6 \) and calculate \( \Delta P = (L \cdot \Delta P)/L = 0.1 \text{ m} \). From Eq. (44) we compute \( Q = 0.2717 \text{ actual m}^3/\text{s} \), thus necessitating the use of \( n = Q_{\text{tot}}/Q = 18.4 \), rounded to 18 parallel cyclones.

One can easily repeat steps 4 and 5 solving for additional values of \( D \), until \( D \) and \( n \) (related to the capital investment cost) are properly balanced with \( \Delta P \) (related to the operating costs). The optimal design will be the one that minimizes the sum of the annualized capital investment and operating costs.

We can use the Leith and Licht (1972) formulation for check through a different method the overall efficiency of the design produced. For the six parallel cyclones solution, Table 1 and Eqs. (5), (8), and (36) yield \( \text{EF} = 0.978 \), \( \gamma = 0.5725 \), \( N = 0.6359 \), and \( A = 0.3306 \). Thus, \( Ad_n = 2.854 \), \( N \ln \sigma_g = 1.075 \), and from the nomograph of Fig. 7 we read \( \text{EF} = 0.935 \).

### Discussion and Conclusions

Cyclones, in contrast to their construction and operating simplicity, are simulated only through complex mathematical models. Even then, the performance of real cyclones involves turbulence, some degree of mixing, some distortion of flow, and may be influenced by the presence of electrostatic effects, the phenomenon of salination, the gas density, and the dust loading, the effects of which are not fully taken into account even by the most prominent models. The above deficiencies are addressed in part through the use of the saltation velocity correlations of Kalen and Zenz (1974), which define the operating region where the particle reentrainment is not excessive and the grade efficiency models are valid. Despite the above difficulties and shortcomings, the selected mathematical models have been verified against experimental data covering widely different conditions and for this reason they find extensive use.

For the analysis of cyclone problems, the above complex grade efficiency models need to be solved simultaneously with the particle-size-distribution relations, and the results combined with the limiting inlet gas velocity and the pressure drop correlations. The graphical solution offered performs the above tasks, allowing fast and rigorous solution to both cyclone performance and cyclone design problems. In doing so, it affords an overview of the cyclone behavior over a wide range of conditions, including an analysis of the so far obscure relationships among overall efficiency, cyclone diameter, and pressure drop. As such it has also an intrinsic educational value that can be important to a designer.

The present paper, in addition to its above stand-alone use for cyclone performance-simulation and design tasks, can find further significant use in the compilation of size-specific particle emis-
mission inventories, the demand for which is rapidly growing due to the newly established detrimental health impacts of fine particles and the introduction of relevant ambient air quality standards. For the latter task, the required nomographs for estimating the overall efficiency graphically are presented in the present paper, while these for estimating the size distribution of penetrating particles graphically can be found elsewhere (Economopoulou and Economopoulos 2001, 2002). Using this output for control devices such as cyclones, planners can easily compile credible PM$_{2.5}$, PM$_{6}$, PM$_{10}$, PM$_{15}$, or PM$_{30}$ inventories from controlled sources (Economopoulou and Economopoulos 2001). The above procedure offers inventory results as reliable as the best models available, with limited effort.

Computers can also be used for the analysis of cyclone problems. However, as relevant programs are not widely available, designers may have to develop their own software and this requires time and skill. In addition, cyclone programs are usually capable of working in one direction, from a given cyclone configuration to performance estimation, but not the other way around when the desirable performance is specified and the cyclone system is to be designed. Finally, most programs are not designed to serve the particular requirements of size-specific particulate emission inventories. The proposed graphical approach addresses the above problems as it is available to—and can be used by—all designers, computer literate or not, it deals as easily with cyclone performance and with design problems, and it serves effectively the requirements of size-specific emission inventories.

Acknowledgment

The first writer is a recipient of a graduate scholarship from the National Scholarships Foundation of Greece.

Notation

The following symbols are used in this paper:

- $A$ = parameter defined by Eq. (36);
- $a$ = inlet height (m);
- $B$ = cyclone dust-outlet diameter (m);
- $b$ = inlet width (m);
- $C$ = parameter defined by Eq. (31);
- $c$ = dust loading (g/m$^2$);
- $D$ = cyclone diameter (m);
- $D_e$ = cyclone gas-outlet diameter (m);
- $D_{max}$ = maximum diameter for cyclone to attain specified $E_f$ (m);
- $d$ = parameter defined by Eq. (13);
- $d_i$ = particle diameter belonging to $i$th size class ($\mu$m);
- $d_m$ = mass median diameter ($\mu$m);
- $E_f$ = cyclone overall efficiency (fractional);
- $E_f$ = grade efficiency (fractional);
- $G$ = configuration factor defined by Eq. (9), with values listed in Table 1 for all “standard” design configurations;
- $g$ = gravitational constant (9.81 m/s$^2$);
- $H$ = cyclone height (m);
- $h$ = cylindrical height of cyclone (m);
- $K$ = parameter in Eq. (18);
- $K_g$ = geometric ratio $a/D$;
- $K_h$ = geometric ratio $b/D$;
- $K_c$ = parameter defined by Eq. (10);
- $K_r$ = geometric ratio $l/D$;
- $K_S$ = geometric ratio $S/D$;
- $K_0$ = parameter defined by Eq. (2);
- $K_1$ = parameter defined by Eq. (3);
- $K_2$ = parameter defined by Eq. (4);
- $L$ = parameter defined by Eq. (47)
- $l$ = natural length of cyclone defined by Eq. (14) (Alexander 1949);
- $M_0$ = cumulative fraction less than size $d_i$;
- $N$ = parameter defined by Eq. (8);
- $N_{H}$ = number of inlet velocity heads defined by Eq. (18) or Eq. (19);
- $n$ = number of parallel cyclones;
- $P$ = gas pressure (atm);
- $PM_i$ = particulate matter with diameter size less than $i$ $\mu$m
- $p$ = overall penetration (fractional);
- $Q$ = cyclone inlet gas flow rate (actual m$^3$/s);
- $Q_{max}$ = maximum cyclone inlet gas flow rate defined by Eq. (38) (actual m$^3$/s);
- $Q_{ax}$ = total gas flow rate to be treated by multiple cyclone system (actual m$^3$/s);
- $S$ = gas outlet length (m);
- $St$ = Stokes’ number defined by Eq. (6);
- $T$ = gas temperature (K);
- $t$ = variate defined by Eq. (25);
- $u_i$ = inlet velocity computed from Eq. (17) (m/s);
- $u_S$ = saltation velocity defined by Eq. (21) (m/s);
- $V_H$ = annular volume around central core, valid for $H - S > 1$;
- $V_{nl}$ = annular volume around central core from $S$ down to natural length $l$, valid for $H - S > 1$;
- $V_S$ = annular volume around exit duct from $S$ up to half the inlet height $a/2$;
- $W_1$ = parameter defined by Eq. (43), with values listed in Table 1 for all “standard” design configurations;
- $W_2$ = parameter defined by Eq. (39), with values listed in Table 1 for all “standard” design configurations;
- $\Gamma$ = parameter defined by Eq. (27);
- $\gamma$ = vortex exponent defined by Eq. (5) (Alexander 1949);
- $\Delta$ = parameter defined by Eq. (28);
- $\Delta P$ = pressure drop (m of H$_2$O);
- $\Delta P_0$ = uncorrected pressure drop for dust loading $c = 0$ (m of H$_2$O);
- $\lambda_1$ = parameter defined by Eq. (29);
- $\lambda_2$ = parameter defined by Eq. (30);
- $\mu$ = gas viscosity (kg/m-s);
- $\rho_g$ = gas density (kg/m$^3$);
- $\rho_p$ = particle density (kg/m$^3$);
- $\rho_w$ = water density (kg/m$^3$); and
- $\sigma_g$ = standard deviation of lognormal distribution.

References
