Seismic traveltime tomography using Fresnel volume approach

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Summary

A Fresnel volume approach is applied to represent wave propagation for seismic traveltime tomography instead of rays. A Fresnel volume is defined as a set of many waves delayed after the shortest traveltime by less than half a period. It is derived by calculating traveltimes both from a source and from a receiver. Tracing rays from sources to receivers is completely avoided. This considerably reduces computational time. We solved the eikonal equation by using a finite-difference method to calculate traveltimes. The advantage of this approach is as follows; First, the frequency of wave can be introduced into analysis. Therefore, we can evaluate the resolution of seismic tomography. Next, The smoothing feature can be naturally introduced. Finally, Fresnel volumes with finite bandwidth considerably reduces the sparseness of ray distribution (data kernel). These advantages make 3-D tomography analysis possible.

Introduction

Conventional seismic traveltime tomography methods are based on the ray approximation which assumes the frequency of waves is infinitely high. The wavelength is thought to be zero. However, actual waves are band-limited. The propagation of actual waves is affected not only by the structures along the ray path as the ray approximation implies, but also by the structures in the vicinity of the ray path.

The area where velocity structures affects a wave with a band-limited frequency is controlled by its frequency. Velocity models divided by cells can not deal with this physical broadness of the wave.

The more physically-realistic representation of wave propagation is to treat a ray path as a beam with finite width. Using a Fresnel volume (Červený and Soars, 1992) is a natural and an effective approach. A Fresnel volume is a set of many rays delayed after the shortest traveltime by less than half the period of wave. The rays in a Fresnel volume are added constructively to form the first-arrival of wave. There have been several studies on the application of Fresnel volumes to seismic tomography since Harlan (1990).

In this study, first, we discussed the characteristics of Fresnel volumes. Next, we formulated the inversion procedure. Then, we investigated the resolution of tomography with respect to the frequency. Finally, we applied this approach to 3-D tomography.

Derivation of Fresnel volumes

Figure 1 shows a schematic representation of a Fresnel volume. The Fresnel volume corresponding to a source (S) and a receiver (R) is represented by the point P that satisfies the following

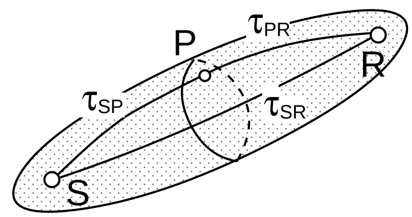


Fig. 1: A schematic representation of a Fresnel volume corresponding to a source S and a receiver R.

equation (Červený and Soars, 1992).

$$\tau_{\mathrm{SP}} + \tau_{\mathrm{PR}} - \tau_{\mathrm{SR}} \leq \frac{1}{2f} \qquad (1)$$

Here, for example, τ_{SR} denotes the traveltime from the source S to the receiver R. f denotes the frequency. This equation means that the point P belongs to the Fresnel volume if the difference between the traveltime of the ray that passes the point P and the shortest traveltime is less than half the period $(T = 1/f)$. The width of a Fresnel volume is inversely proportional to the square root of the frequency.

We used the following procedure to calculate Fresnel volumes.

- First, calculate the traveltime from a source to each grid point P, τ_{SP}, in the area of interest.
- Next, calculate the traveltime from a receiver to each point P, τ_{RP}.
- Then, add together the traveltimes from the source, τ_{SP}, and those from the receiver, τ_{RP}. Because the wave propagation holds reciprocity ($\tau_{\mathrm{RP}} = \tau_{\mathrm{PR}}$), the result ($\tau_{\mathrm{SP}} + \tau_{\mathrm{PR}}$) shows the traveltimes from the source to the receiver through the point P (Harlan, 1990; Matsuoka and Ezaka, 1992).
- After subtracting the shortest traveltime, τ_{SR}, extract the Fresnel volume by applying the threshold of delay time, $1/2f$.

We used a finite-difference solution of the eikonal equation (Vidale, 1988; Qin *et al.*, 1992) to calculate traveltimes. This method is fast and efficient in computation.

We represent a Fresnel volume as weight values, ω, given at each grid point P. Our intension was to expand a ray to a beam with finite width. If the traveltime delay at a point P is small, the point is in the vicinity of the axis of Fresnel volume, namely the ray

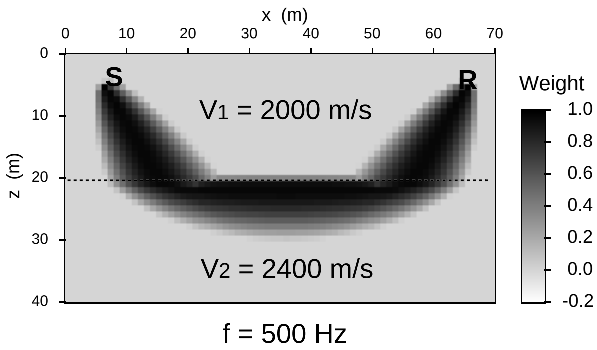


Fig. 2: A Fresnel volume of a head wave with the frequency of 500 Hz in a two-layered medium. The velocity of the upper and bottom layer is 2000 m/s and 2400 m/s, respectively.

path of the shortest traveltime. It is quite likely that such points give significant effects on wave propagation. Therefore, we expressed the weight values as a monotonously decreasing function with respect to the delay. In this study, we used a linear weighting function described below;

$$\omega = \begin{cases} 1 - 2f\Delta t, & (0 \leq \Delta t \leq 1/2f) \\ 0, & (1/2f \leq \Delta t) \end{cases} . \quad (2)$$

Here,

$$\Delta t = \tau_{\mathrm{SP}} + \tau_{\mathrm{RP}} - \tau_{\mathrm{SR}}. \quad (3)$$

To obtain Fresnel volumes, ray-tracing from sources to receivers are completely avoided. This significantly reduces the computational time and makes 3-D calculation possible.

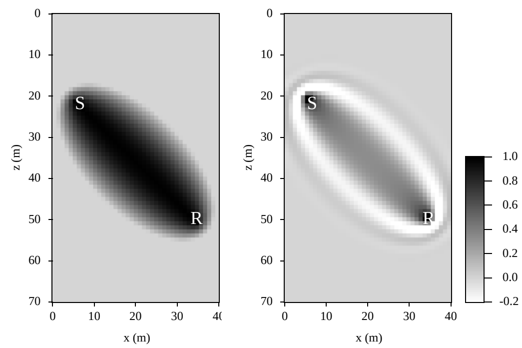
Figure 2 shows an example of a Fresnel volume of a head wave represented as weight values.

Fresnel volume approach and wave theory

In the full-wave inversion based on the acoustic wave-equation, the error function to be minimized is defined as the total power of the residuals of waveform. The gradient, $\gamma(\boldsymbol{x})$, of the error function with respect to the velocity is calculated by correlating the forward-propagated wavefield and the backward-propagated wavefield of the residuals (Tarantora, 1984; Luo and Schuster, 1993).

Figure 3 shows the relation between the Fresnel volumes and the full-wave inversion. Figure 3(a) shows the Fresnel volumes represented by a linear weighting function in a homogeneous medium of 2000 m/s. The frequency of wave is 150 Hz. Figure 3(b) shows the normalized gradient of the error function of the full-wave inversion, $\gamma(\boldsymbol{x})$. The source is a Ricker wavelet of 150 Hz.

The Fresnel volume covers almost the same area as the gradient, although the distribution of the weight is different. Yomogida (1992) theoretically derived the Fréchet derivative for phase by using Rytov approximation of wave-equation. The result shows that the derivative is zero in the center and has high value at the edges of a Fresnel volume. The result shows that Fresnel



(a) (b)

Fig. 3: Relation between the Fresnel volumes and the full-wave inversion. (a) The Fresnel volume expressed by using the linear weight function and (b) the normalized gradient of the error function in the acoustic full-wave inversion.

volumes coincide with the area where a band-limited wave is affected by the medium. By selecting an appropriate weighting function, Fresnel volume can represent the wave propagation with finite frequency derived from the wave theory. The advantage of the Fresnel volume approach is that it does not require the computation time compared to other approaches based on the wave theory. The Fresnel volume approach is intermediate between the ray approximation and the wave theory.

Inversion Procedure

Vasco *et al.* (1995) used coarse cells to represent slowness structures. We provide the slowness on the fine grid points which are the same as those used to calculate the traveltime and the Fresnel volumes.

The updating slowness, ΔS_j, on the grid points is obtained by the following equation;

$$\frac{\Delta S_j^k}{S_j^{k+1}} = \sum_{i=1}^{N} \frac{\omega_{ij} \Delta T_i}{T_i^{\mathrm{obs}}} \Bigg/ \sum_{i=1}^{N} \omega_{ij}. \quad (4)$$

Here, i, j and k are the suffixes to denote the data, the grid point and the iteration, respectively. ΔT is the traveltime residual represented by the following equation.

$$\Delta T = T^{\mathrm{obs}} - T^{\mathrm{cal}} \quad (5)$$

T^{obs} and T^{cal} denote the observed traveltime and the calculated traveltime with respect to the current model. Equation (4) is an expansion of SIRT with respect to the ray with finite width. The slowness of the grid point within the Fresnel volume is updated. Therefore, the physical broadness of wave can be evaluated exactly in the inversion.

Resolution of tomography

The wavelength determines the resolution of seismic tomography if the spatial sampling is high enough. We discussed the

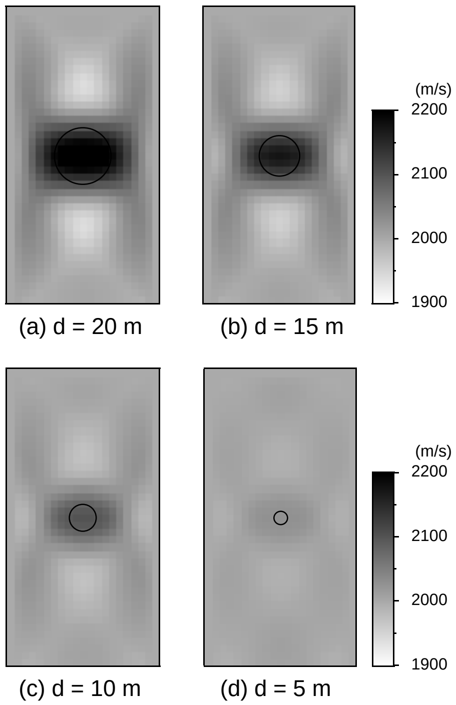


Fig. 4: The reconstructed tomogram by using the Fresnel volume method. The diameter, d, of the anomaly (indicated by an open circle) is (a) 20 m, (b) 15 m, (c) 10 m and (d) 5m, respectively.

effect of frequency on the resolution of tomography by using the Fresnel volume approach.

The velocity structure used is a high-velocity anomaly (2400 m/s) model at the center of a homogeneous background (2000 m/s). The size of the area of interest is 50×100 m. The diameters, d, of the anomalies are 20 m, 15 m, 10 m and 5 m. The frequency of wave is 400 Hz. Therefore, d/λ is 4, 3, 2 and 1, respectively. Forty-one sources and 41 receivers are placed on each side of the model. To make a traveltime dataset, we picked the first arrival of the calculated waveforms at the receivers. The waveforms are obtained by solving the 2-D acoustic wave-equation using the finite-difference method.

Figure 4 shows the reconstructed tomogram by using the Fresnel volume method. The Fresnel volume method generates a smooth tomogram because of its averaging nature. The horizontal resolution of 9.7 m and the vertical resolution of 11.2 m are obtained by the theoretical resolution of crosswell tomography proposed by Schuster (1995). From the figure, we conclude that the resolution of the traveltime tomography is about double to the wavelength (10 m) in this case. The Fresnel volume method generates tomograms that reflects the resolution accurately.

The actual wave contains various frequency components. After separating them by band-pass filters, a multi-resolution tomography analysis is possible by using the Fresnel volume approach. If we start with low frequency to make a tomogram and then move

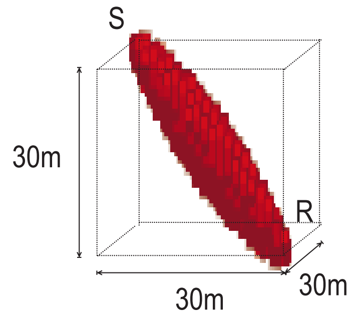


Fig. 5: A Fresnel volume with the frequency of 150 Hz in a 3-D homogeneous volume of 2000 m/s.

to higher frequency, we can get more focused tomogram with less artifact.

Improving sparseness of ray density

Figure 5 shows an example of the Fresnel volume in a 3-D volume. The source and the receiver are 30 m apart in x, y and z directions, respectively. Consider that the target cubic volume is divided into small cubic cells of 1 m × 1 m × 1 m by grid points. The Fresnel volume of 150 Hz covers 783 grid points out of 29791 (31×31×31) grid points. On the other hand, the ray passes through only 30 cells out of 27000 (30×30×30) cells. Therefore, the Fresnel volumes can considerably reduce the sparseness of ray distribution. That makes inversion process stable.

Application to 3-D tomography

The 3-D tomography is a quite attractive method to image underground structures. However, it is still quite costly both in data acquisition and in analysis. Quite many sources and receivers must be arranged to cover the target volume. For all that efforts, the ray distribution is so sparse that we can get a rough image represented by coarse cells.

Fresnel volume approach reduces sparseness of ray distribution (data kernel) considerably and makes the inversion process stable. In addition, the computation is fast and efficient compared to other ray-tracing methods. Therefore, we applied the Fresnel volume method to the 3-D tomography.

Figure 6 shows the result of the synthetic study of 3-D tomography. Figure 6(a) and (b) shows the model and the location of the sources and receivers for the synthetic data. Figure 6(c) and (d) show the result of the 3-D tomography. In Figure 6(d), the upper layer is removed to show the layer boundary. The strike and the dip of the layer are reconstructed accurately.

Conclusions

In this study, we introduced a Fresnel volume approach into tomography analysis. The approach has following advantages;

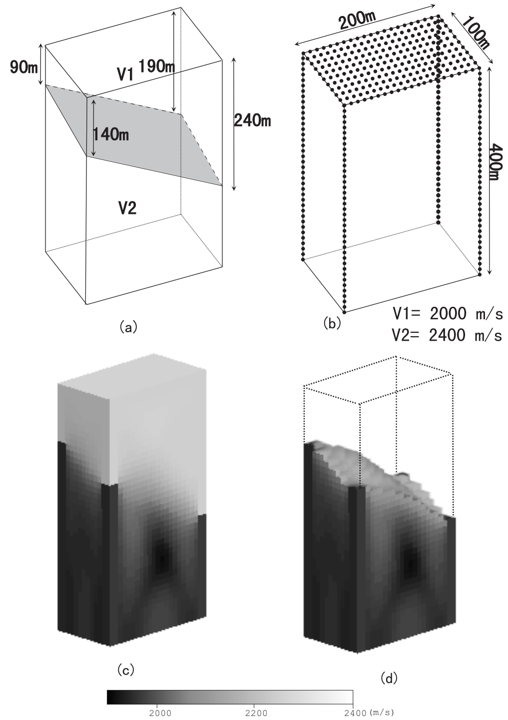


Fig. 6: Synthetic study of 3-D tomography. (a) The model, (b) the location of sources and receivers and (c), (d), the result of the 3-D tomography.

1. Traveltime calculation based on the eikonal equation is fast and efficient.

2. Ray-tracing is completely avoided.

3. The effect of frequency on tomogram can be considered.

4. The smoothing effect is naturally introduced.

5. The sparseness of the ray distribution is considerably reduced. It makes the inversion process stable.

We investigated the resolution of tomography with respect to the frequency. The result shows the resolution of traveltime tomography is comparable to the wavelength. From the result of the 3-D tomography, we conclude the method is suitable for the 3-D tomography.

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