The Flaw of Averages
Why We Underestimate Risk in the Face of Uncertainty
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Introduction
"Our culture encodes a strong bias either to neglect or ignore variation. We tend to focus instead on measures of central tendency, and as a result we make some terrible mistakes, often with considerable practical import." - Stephen Jay Gould, naturalist, 1941-2002

The measure of central tendency that Gould refers to is typically the average, also known as the expected value, and the mistakes he warns of result from a common fallacy as fundamental as the belief that the Earth is flat. It permeates planning activities in business, government and the military. It helped mask the recent subprime mortgage fiasco until it became a world crisis, and it will plague those trying to clean up the mess. It's even enshrined within accounting codes.

It can be called the Flaw of Averages. It states, in effect: Plans based on average assumptions are wrong on average.

An apocryphal example concerns the non-swimming statistician who drowned while fording a river that was, on average, only three feet deep - but in one spot he stepped into a gulch that was 15 feet deep.

In everyday life, the Flaw of Averages ensures that plans based on average customer demand, average completion time, average interest rate and other uncertainties are below projection, behind schedule and beyond budget.

Give Me a Number
To understand how pervasive the Flaw of Averages is, consider the hypothetical case of a marketing manager who has just been asked by his boss to forecast demand for a new-generation microchip.

"That's difficult for a new product," responds the manager, "but I'm confident that annual demand will be between 50,000 and 150,000 units."

"Give me a number to take to my production people," barks the boss. "I can't tell them to build a production line with a capacity between 50,000 and 150,000 units!"

The phrase, "Give me a number," is a dependable leading indicator of an encounter with the Flaw of Averages, but the marketing manager dutifully replies, "If you need a single number, I suggest you use the average of 100,000."

The boss plugs the average demand, along with the cost of a 100,000-unit production line, into a spreadsheet model of the business. The bottom line is a healthy $10 million at that production level, which he reports as the projected profit. Assuming that demand is the only uncertainty and that 100,000 is the correct average (or expected) demand, then $10 million must be the average (or expected) profit. Right?

Wrong. The Flaw of Averages ensures that on average, profits will be less than the profit associated with the average demand.

Why? If the actual demand is only 90,000, the boss won't make the projection of $10 million - profit will fall short of that because not as many units are being produced and sold. If demand is $80,000, the results will be even worse. That's the downside.
On the other hand, what if demand is 110,000 or 120,000? Unfortunately, that demand exceeds production capacity and the company can only sell 100,000 units. So profit is capped at $10 million. There's no upside to balance the downside. The average profit, therefore, is lower than the profit associated with the average demand.

**Behind Schedule**

But why are things behind schedule on average? Consider a software project that requires 10 separate subroutines to be developed in parallel. The boss asks the programming manager of the first subroutine how long development will take.

"I'm confident it will take between three and nine months," replies the programming manager.

"Give me a number," says the boss. "I have to tell the chief operating officer when we'll be operational!"

"Well," says the programming manager, "on average, programs like this take about six months. Use that if you need a single number."

For simplicity of argument, suppose the boss has similar conversations with each of the nine remaining programming managers. The durations of all the subroutines are uncertain and independent, and they're expected to range between three and nine months with an average of six months. Because the 10 subroutines are being developed in parallel, the boss now goes to the COO and happily reports that the software is expected to be operational in six months.

Imagine there's a 50% chance of hitting the average time in each case. Getting all right is analogous to flipping a coin 10 times and having it come up heads each time. The odds are less than one in 1,000 of that occurring.

**Being Over Budget**

And why is everything over budget on average?

Consider a pharmaceutical firm that distributes a perishable antibiotic. Although demand fluctuates, the long-term average is a steady five cartons of the drug per month. A new VP of operations has taken over the distribution center. He asks the product manager for a forecast of next month's demand.

"Demand varies," responds the project manager, "but I can give you an accurate distribution - that is, the probabilities that demand will be zero, one, two, three and so on." The product manager, apprehensive about his new boss, is relieved that he can provide such complete information in his first professional interaction.

"If I'd wanted a distribution, I would have asked for a distribution," snaps the boss. "Give me a number so I can calculate operating costs." Eventually they settle on the time-honored tradition of representing the uncertainty by its average. Armed with the accurate average demand of five cartons per month, the boss now proceeds to estimate inventory operating costs, calculated as follows:

- If monthly demand is less than the amount stocked, the firm incurs a spoilage cost of $50 per carton of the perishable drug.
- On the other hand, if demand is greater than the amount stocked, the firm must airfreight the extra cartons at an increased cost of $150 each.

A quick calculation indicates that if five cartons are stocked, and the demand happens to come in right at its average of five, then there will be neither spoilage nor airfreight costs. Thus, the boss reasons, the average inventory operating cost will be zero. Right?

Wrong! If demand is below average, the firm gets whupped upside the head with spoilage costs, whereas if demand is above average, the firm gets whupped on the other side of the head with airfreight costs. No negative costs exist to cancel out the positive one - so, on average, the cost will exceed the cost associated with the average demand.

**A Problem of Dilbertian Proportions**

When managers ask for a forecast, they're really asking for a number, which involves the Flaw of Averages.
For example, the product manager of the new microchip facility provided the correct forecast of average demand to his boss. But the boss turned around and used that single number to incorrectly forecast average profit. In that case - and with the programming managers and pharmaceutical firm, where again correct averages led to incorrect forecasts - the bosses will ultimately claim they got bad forecasts from their subordinates and will end up punishing them for what was in fact the correct average. This is a problem of Dilbertian proportions.

In his book *A Whole New Mind*, author Daniel Pink predicts the ascendance in the economy of right-brained, big-picture thinking relative to left-brain analysis. The subjects of statistics and probability have traditionally been the domain of the left. But in fact the Flaw of Averages often arises due to the left brain’s stubborn insistence on a single precise answer. If anything, the right side of the brain is better equipped to interpret the patterns inherent in uncertainty.

**Uncertainty and Risk**
In assessing forecasts, we’re often concerned with risk and uncertainty. The terms uncertainty and risk, in fact, are often used interchangeably, but they shouldn’t be.

For example, a wino and Bill Gates might attach very different risks to the same uncertainty. Uncertainty is an objective feature of the universe, whereas risk is in the eye of the beholder.

Is there a risk that XYZ Corporation stock will go down tomorrow? Not for someone who sold XYZ short. That individual will make money only if XYZ goes down. He or she will suffer a loss only if the stock goes up. The uncertainty is the volatility of XYZ stock - the risk depends on whether a person is long or short.

Risk is in the eye of the beholder. It reflects how uncertain outcomes cause loss or injury to a particular individual or group. Risk attitude measures the willingness to incur risk in the quest of reward.

**An Uncertain Number Is a Shape**
Statisticians often describe a numerical uncertainty with the scary words, random variable. But it’s better to call it simply, uncertain number. Think of it as a shape - the shape of things to come. Statisticians call it the probability distribution, or just distribution for short.

So it’s possible to take the previous examples, such as the uncertainty of microchip demand, and produce a chart - for example, a histogram or bar graph - showing the probability of demand for various levels between 50,000 and 150,000 units. The height of the bars indicates the relative likelihood that the specific demand will occur, and all the values add up to 100%.

We need to stop thinking of uncertainties as single numbers - the average - and instead begin thinking of them as shapes, or distributions. And to deal with those distributions, we need to take advantage of modern computers for probability management.

**How Probability Management Works**
Probability management can be viewed as a data management system in which the entities being dealt with are uncertainties - that is, probability distributions. The central database for such efforts is a scenario library containing thousands of potential future values of uncertain business parameters. The library exchanges information with desktop distribution processors that do for probability distributions what word processors did for words and what spreadsheets did for numbers.

The practice of probability management is still in its infancy, but it’s already being applied at organizations such as Shell, Merck and Olin. At these and other organizations, managers across the enterprise are beginning to visualize, communicate and interact coherently with risk and uncertainty in ways that were unimaginable a few years ago.

It had its roots in Monte Carlo simulation. To understand Monte Carlo simulation, think of how you give a ladder a good shake before you climb up it to paint the side of your house. By bombarding the ladder with random physical forces, you simulate how stable it will be when you climb on it. You can then adjust accordingly to minimize the risk that it will fall down with you on it.

Monte Carlo simulation is a computational technique similar to the shaking of the ladder, which can test the stability of uncertain business plans, engineering designs or military campaigns by bombarding a model of the
business, bridge or battle with thousands of random inputs, while keeping track of the outputs. This allows you to estimate the chance that the business will go bust, the bridge will fall down or the battle will be lost.

Simulation does for uncertainty what the light bulb does for darkness. It doesn't eliminate uncertainty any more than light bulbs prevent the sun from setting. But simulation can illuminate the Flaw of Averages just as the bulb illuminating your basement stairway reduces the odds that you'll break your neck.

Monte Carlo simulations and other methods of probability management show great promise in illuminating the Flaw of Averages, but much work remains to be done. Eventually it should be possible for firms to create consolidated, auditable risk statements that may provide more warning about future financial upheavals of the kind we had in 2008.

Conclusion
When someone says, "Give me a number," that should be a warning that you could fall prey to the Flaw of Averages. Instead, think about distributions and probabilities, and how you can manage them.

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About the author: Sam Savage is a consulting professor of management science and engineering at Stanford University.

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