Some Applications of the Barlat 1991 Yield Criterion

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Abstract
Nowadays the accuracy of (FEM) analyses of forming processes is to a large extent determined by the assumption of realistic constitutive equations for the description of the material and its interface with tooling. Concerning aluminium alloys there are reasons for attention to the first topic: these alloys often show significant anisotropy and in general have the reputation of being troublesome in analyses of forming processes. Through some applications, it will be shown that a yield criterion proposed by Barlat et al. (1991) gives good results. Last criterion is based on the generally accepted framework of a yield surface and associative flow. It thereby preserves simple operational use.

Keywords: aluminium, yield, anisotropy

1 Introduction
This paper will concentrate on two anisotropic 6-D yield criteria and their validation. The final goal is accurate prediction of plastic flow and process forces in forming processes. Accuracy will depend on the accumulation of errors obtained from measurement of material properties, yield criteria, hardening assumptions and the applied model. These are not independent: e.g. extreme accuracy in measurement of material parameters will in general not be useful if the yield criterion is an order of magnitude less accurate.

For a general 6-D stress situation three anisotropic criteria are available from literature [1..3]. For the limiting but generally present case of orthotropy two [2,3] of these are practically equivalent so that the yield criterion presented in [3] is neglected in this work. Isotropic work-hardening is assumed throughout this paper so that the results are an indication for the usefulness of a yield criterion combined with the assumption of isotropic work hardening. The criteria are validated through tensile and compressive testing, pure bending and cup drawing of aluminium alloys.

2 Two 6-D anisotropic yield criteria
Most often the behaviour of aluminium alloys deviates significantly from isotropy. To accommodate for this, the obvious solution is the use of the classical Hill (1948) [1] yield criterion. It states that (Eq. 1) :

$$\phi = F (\sigma_y - \sigma_z)^2 + G (\sigma_z - \sigma_x)^2 + H (\sigma_x - \sigma_y)^2$$

$$+ 2L \sigma_y \sigma_z + 2M \sigma_z \sigma_x + 2N \sigma_x \sigma_y = 2 \sigma^2$$

should be fulfilled at yield where F,G,H,L,M and N are material parameters. This yield criterion is standard to most commercial FEM packages. An alternative, relatively recent, criterion proposed by Barlat et al. (1991) [2] states (Eq. 2):

$$\phi = \frac{F (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2}{3} + \frac{(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2}{54}$$

$$I_2 = \frac{(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2}{54}$$

$$I_3 = \frac{(\sigma_x - \sigma_y) (\sigma_x - \sigma_z) (\sigma_y - \sigma_z) + 1ghFGH}{6}$$

$$\theta = \arccos \left( \frac{I_2}{I_3} \right)$$

$$\phi = 2 \sigma - m = (3I_2)^{m/2}$$

where a,b,c,f,g,h are the weight factors applied to the corresponding components (A,B,C,F,G,H) of Bishop/Hill stress. This yield criterion is an extension to Hosford's 1972 [4] isotropic yield criterion. For the accompanying flow rule etc. the reader is referred to [2] and [5]. Main feature of the yield criterion is that by an appropriate choice of the parameter m, the amount of curvature of the yield function near points of uniaxial tensile or compressive stress can be influenced. Higher values of m give yield criteria resembling a 'round off' Tresca yield criterion like found in experiment [8] at small strain (1%).

3 Material characterisation

3.1 Sheet
Aluminium alloy sheet of 3 mm thickness is used for experiments. R-values and flow curves are given in table 1 and fig. 1. Flow curves in thickness direction are measured using $12 \times 12$ mm 'through thickness' compression test specimens machined from a pile of 4 sheets glued together.
Table 1: Measured R-ratios for sheet material at 0, 45 and 90 degrees to the rolling direction.

<table>
<thead>
<tr>
<th>Material</th>
<th>( R_0 )</th>
<th>( R_{45} )</th>
<th>( R_{90} )</th>
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<tr>
<td>AA-6351</td>
<td>0.50</td>
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<td>AA-6351-O</td>
<td>0.55</td>
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![Flow curve measured in 0° direction compression tests (duplicate).](image)

4. Validity of yield criteria

4.1 Flow curves

The constants of the yield criteria are fitted using the measured \( R_0 \), \( R_{45} \) and \( R_{90} \) values. Using the assumption of work hardening, a yield criterion and the flow curve in 0° direction, it is possible to calculate the other flow curves. Fig. 4 gives the results for the sheet alloys. Fig. 5 visualises the difference between the two yield criteria.

![Fig. 4: Measured flow curve in rolling direction and flow curves calculated with a yield criterion and the work hardening assumption. Top: Barlat 1991 criterion with \( m = 12 \); bottom: Hill 1948 criterion. Compare with fig. 1.](image)

![Fig. 5: Visualisation of yield criteria in \( \sigma_{90} - \sigma_{0} \) principal stress space (\( \sigma_1 = 0 \)). Material: 6351.](image)
4.2 Pure bending

Bending of sheet is performed using an apparatus [7] for bending with a pure moment. Bending is continued up to a given bending moment. After unloading the outer radius of the specimen is measured. From a number of experiments the moment-radius relation is known.

Implicit FEM-simulation is used to calculate the same relation from the flow curve measured in the direction of rolling (fig. 1) and $R_0$ and $R_{00}$. Young's modulus and Poisson ratio are assumed to be 70 GPa and 0.345 respectively. The Barlat yield criterion is implemented into FEM program ABAQUS through a user subroutine UMAT [5]. The FEM mesh consists of 100 quadratic elements. In the simulation only a small portion of the arc is simulated. Kinematical boundary conditions are chosen such that loading occurs with a pure moment. Fig. 6 schematically shows the mesh and the imposed boundary conditions. Fig. 7 shows an example of the calculated stresses for a particular situation.

**Fig. 6:** FE mesh (schematically); Diagram: Bending moment $M^* = M / C b_0 s_0^2$ versus $s_0^2/\rho_m^2$, $\rho_m$ is the radius of the mid-plane; curve: elastic/rigid plastic analytical model [7] assuming constant thickness; dots: FEM; Von Mises: $E = 67 \text{ [GPa]}, \nu = 0.365$; $C = 126 \text{ [MPa]}, n = 0.042$, $\varepsilon_0 = 0.0001$ (01$ = C (\varepsilon + \varepsilon_0)^m$ $s_0$ initial thickness, $b_0$: initial width.

Results of calculations using the Hill (1948) criterion and the Barlat (1991) criterion, with $m = 8$ and 12, are given in fig. 8. Not shown is that the use of the Von Mises yield criterion and the work hardening assumption leads to an overestimation of the bending moment by some 10 to 15%. This last result is also found for a soft aluminium alloy [7]. Good results are obtained for both anisotropic criteria for alloy 6351. For 6351-0 the validity of the work hardening assumption has to be doubted since results are worsening for shaper bends.

**Fig. 7:** Example of calculated principal and hydrostatic stresses over the thickness $s$ (Barlat criterion, $m = 12$).

**Fig. 8:** Measured (dots) and calculated bending moment versus initial thickness/outer radius (unloaded); open dots and dashed curves: bending axis parallel to RD; solid dots & curves: bending axis perpendicular to RD; for 6351-0 two curves happen to be approximately equal; Specimen dimensions: (thickness x width) 6351: 3.03 x 95; 6351-0: 3.02 x 96.

4.3 Cylindrical cup drawing

The profile shown in fig. 2 is selected for its high in-plane anisotropy so that the performance of the two yield criteria can easily be verified using the cup earing profile. Fig. 9 shows the functions $R(a)$ and $\sigma(a)/\sigma_0$ for two yield criteria. Note the large variation in $\sigma(a)/\sigma_0$ from 0.9 to 1.3 for the Hill (1948) criterion.

**Fig. 9:** Calculated functions $R(a)$ and $\sigma(a)/\sigma_0$ for Hill (1948) and Barlat (1991) criterion.
Blanks having 18 mm diameter and 0.73 mm thickness are ultra precision machined from the middle of the strip section of the profile. The die and punch diameter equal 12 and 10 mm. The blank holder force, 0.3 KN, is applied using dead weight. Teflon foil is applied as lubricant at the blank/blankholder and blank/die interface. The punch/die interface is non-lubricated. After drawing the cup height is measured for comparison with simulation. Three dimensional simulation is performed using the FEM program ABAQUS Explicit. The blank is modeled using 2 layers of linear reduced integration brick elements. Hereby the bending and unbending at the die radius is taken into account. Fig. 10 shows an undeformed and deformed mesh. The coefficient of Coulomb friction is assumed to equal 0.06 at the lubricated sites and 0.15 at punch-blank interface. In Fig. 11 the measured and calculated cup height is compared. From this figure it is clear that the Barlat criterion with \( m \) equal to 12 gives best results. The result based on the Hill (1948) criterion overestimates the earing tendency by a factor two.

**5 Conclusions**
- The Hill (1948) criterion overestimates the differences in flow stress for aluminium alloys. The Barlat 1991 criterion performs better here. Only for alloy 6351 a large difference in yield stress is observed (in thickness direction). When fitted to \( R_0 \) and \( P_{60} \) both criteria give an opposite yield stress difference here (figs. 1 and 4).
- For the bending experiments good results are obtained for both anisotropic criteria. However in the bending experiments, the assumption of work hardening only holds for alloy 6351. For the alloy 6351-O, deviation between bending experiment and simulation increases with the amount of plastic work. This suggest that the work hardening assumption is not accurate in this case. The work hardening assumption seems more critical when the hardening of the material is strong.
- Application of the Barlat (1991) criterion gives good results for the prediction of earing in cup drawing of an aluminium alloy showing large in-plane anisotropy.

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**References**

**Appendix**

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**Table A.1:** Material parameters (Eq. 3) characterising orthotropy. Yield criterion: Barlat (1991).