THE GRADUAL CHANGES OF SYNODIC PERIOD OF THE MOON PHASE

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Abstract. The length of synodic period of the moon with the same Metonic group is studied through the period of the two subsequent conjunctions. We select the Metonic group of lunation when the conjunction occurs around perihelion, aphelion, spring and autumnal equinox, winter and summer solstices. The regularity of synodic period in internal Metonic group has been found for the position of the Sun closed to equinox, solstices, perihelion, and aphelion. The position of the Sun is reflection of the speed of the earth moves around the Sun. It will affect the length of synodic period as well as the range of synodic period. The Moon orbital complexity is reflected to the pattern of regularity in the plot of synodic period againts ILn for each ILVn. The gradual change of synodic period for all group of ILVn is clearly composed of regularity of each ILVn.

Keywords: Synodic period, moon phase, lunar calendar, Meton

1 Introduction

Among calendar system, one constructed based on astronomical phenomena, such as Hijriah calendar based on visibility of thin lunar crescent after conjunction which is called hilal. The new month in Hijriah calendar begins when hilal appears (can be seen by naked eye) after sun set time. The average of Hijriah month or Islamic month closed to synodic month, which defined the mean time interval between two consecutive conjunction of the Moon and the Sun.

One month in Hijriah calendar consist of 29 or 30 days, and the average synodic month is 29.530589 days. More precise formula based on the lunar theory [3] $29.5305888531+0.0000002161T-3.64\times10^{-10}T^2$ where $T = (JD-2451545.0)/36525$, JD=Julian Day Number [4]. Stephenson and Baolin [19] found the range of synodic month between 29.2679 days and 29.8376 days. Espenak and Meeus [7] found the shortest and longest synodic period of the moon 29.26574 days (6 hours 21 minutes 23 seconds shorter than the mean value) and 29.84089 days (7 hours 26 minutes 50 seconds longer than the mean value).

The position of the Sun in the sky repeats every 365 days and the changes appearance of the Moon repeats every 28 days (some very old and very young crescent may not counted). Roughly recurrence the appearance of the Sun and the node of the Moon is (365/28) years, ~13 years (the reality will be longer than 13 years). The eclipse recurrence called Saronic cycle, equal to 223 lunation or equal to roughly 18 years. The Saronic cycle shows the changes of various angular diameter of the moon as it is shown various type of solar eclipses (partial, annular or total) subsequently in series of eclipses in a Saros number. Espenak [5] and Espenak & Meeus [7] found relationship between some periods of Eclipse (P) and the Saros number such as Semester (P = 99 lunation; ~107.43 Draconic months); Tritos (P = 135 lunation; ~146.5 Draconic months); Meton (P= 235 lunation; ~255 Draconic months), Inex (P = 358 lunation; ~388.5 Draconic months) and Exeligmos (Triple Saros) (P = 669 lunation; ~726 Draconic months). Meeus [10] found longer period 372 years or 4601 lunation about 4993 Draconic months. Another moon phase cycle of 251 lunation (~269 anomalistic months) associates with the position of moon phase (full moon or new moon etc) when the moon at perigee or apogee [16].

The phase of new moon occurs in a moment of solar eclipse. There is a cycle of the same moon phase occurs on almost the same date every 99 lunation (~8 years), 235 lunation (~19 years) and 4601 lunation (~372 years). We use 235 lunation as basis for grouping the data for the purpose a study on determining the beginning of new month in Islamic lunar calendar (Hijriah calendar).

In daily practice, tabulation of Hijriah calendar, a month consists of 29 or 30 days. The differences between the synodic month and the length synodic period are small, within 0.2 days up to 0.7 days. The moon orbit inclined about 5^o to the ecliptic, it implies during the moon's revolution around the Earth, the moon moves in the possible area with extreme declination -28.5^o South and $+28.5^o$ North. The appearance of moon crescent is also affected by the position of the Sun in the sky, it can be compared when the conjunction happens during the position of the Sun at extreme declination such as solstices or when the earth closed to the perihelion and aphelion passage. Due some disturbances on Moon's orbit revolve around the earth and geometry between the earth, the moon and the sun cause some differences on the length of synodic month.

2 The Data of Synodic Period of the Moon

Meeus [11] developed algorithm of moon phases based on lunar theory Chapront's ELP – 2000/82 and Solar theory Bretagnon and Francou [1]. The Algorithm is used to calculate the length of synodic month, the interval between the two new moons. We calculate the data of all synodic period from 622 AD to 3000 AD, a part of the data provided in Table 1.

Table 1. A part of the data new moon. Y: Gregorian year, M: month, D: date, h: hour, m: minute, s: second, ILn: Islamic Lunation Number, ILVn: Islamic Lunar Variant Number, MH: Hijriah month, YH: Hijriah year, $P_{synodic}$: synodic period (days).

Y	M	D	h	m	s	ILn	ILVn	MH	YH	$P_{synodic}$
2008	Jan	08	18	37	03	17137	217	Muharram	1429 H	29,67177
2008	Feb	07	10	44	24	17138	218	Safar	1429 H	29,56232
2008	Mar	08	00	14	08	17139	219	Rabi'ul Awal	1429 H	29,44525
2008	Apr	06	10	55	17	17140	220	Rabi'ul Akhir	1429 H	29,34930
2008	May	05	19	18	17	17141	221	Jumadil Awal	1429 H	29,29469
2008	Jun	04	02	22	38	17142	222	Jumadil Akhir	1429 H	29,28887
2008	Jul	03	09	18	37	17143	223	Rajab	1429 H	29,32911
2008	Ags	01	17	12	32	17144	224	Sya'ban	1429 H	29,40662
2008	Ags	31	02	58	04	17145	225	Ramadhan	1429 H	29,50993
2008	Sep	29	15	12	22	17146	226	Syawal	1429 H	29,62610
2008	Oct	29	06	13	57	17147	227	Dzulkaedah	1429 H	29,73663
2008	Nov	27	23	54	41	17148	228	Dzulhijjah	1429 H	29,81098
2008	Dec	27	19	22	30	17149	229	Muharram	1430 H	29,81442

Table 1 shows an example of the Moon phase in 2008 (West Indonesian Time = UT + 7 hours). Islamic lunation number (ILn) and Islamic lunar variant number (ILVn) are calculated follow the definition in Raharto [13].

3 Metonic Group of Hilal

The length of Metonic cycle is 235 lunation and it is equal to 19 mean Solar years. The moon phases in Metonic period will occur in the same or almost the same date of Gregorian calendar. Hilal visibility occurs after conjunction of the Moon and the Sun, and then it is useful to make a groups of hilal according to the Islamic lunar variant number (ILVn) in order to know the characteristic of hilal. Further more, it is easier to find member of groups of hilal when the Sun in equinox and solstices, as well as when the earth at perihelion and aphelion.

We use all calculated data of hilal from ILn 15000 to ILn 20000 presented in Figure 3, 4 and 5. The data of hilal with ILn belongs to ILVn 223, ILVn 87 and ILVn 186 are used to study hilal closed to aphelion. For hilal closed to perihelion, we use ILn which are member of ILVn 155, ILVn 19 and ILVn 217. In order to understand the characteristic hilal closed to Vernal Equinox (March equinox) we use ILn which belong to ILVn 71, ILVn 170 and ILVn 34. For hilal closed to Autumnal Equinox (September equinox) we use all ILn which are member of ILVn 201, ILVn 65 and ILVn 164. For hilal closed to Summer Solstice (June solstice) we use ILn belong to ILVn 37, ILVn 235 and ILVn 99. The data of hilal belongs to ILVn 204, ILVn 68 and ILVn 167 are used to study hilal closed to Winter Solstice (December solstice).

For one year period, the synodic period of each lunation is not the same. In order to see the characteristic of the changes of synodic period we plot the synodic period againts ILn, the plot presented in Figure 1. For long period, the periodicity of synodic period shows fairly regular pattern (see Figure 1). The regular pattern is also shown for longer period than provided in Figure 1. It can be seen in Figure 1 that there is a gradual changes of synodic period. The changes has a regular pattern from shorter to longer period and again return to shorter period.

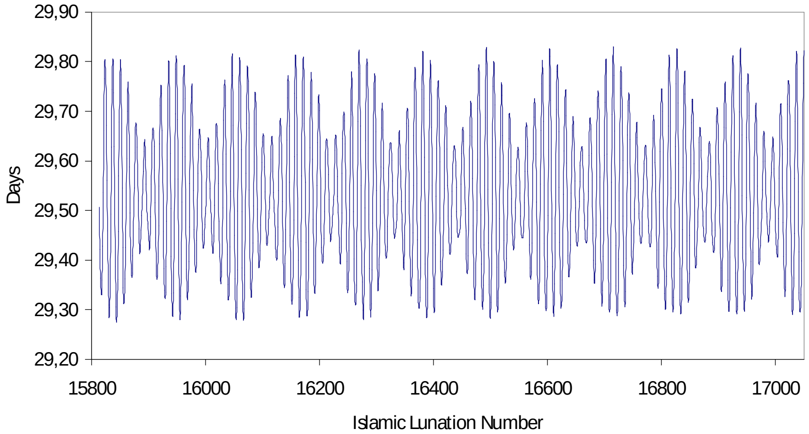

Figure 1. Synodic period vs ILn from January 1901 to December 2000

In order to find the periodicity of the data, we apply window-CLEAN algorithm (WCA) and Lomb-Scargle spectral analysis (LSA), and result provided in Figure 2. There are two independent cycle on synodic period of the moon in Figure 2 represented by two peaks, the first peak, P1 = 12.3685 and the second peak, P2 = 13.94444. The first peak periodicity may associate to Saronic periodicity (SP) with unit of "SP = (P1* 29.53059 days) = 365.2429024 days" closed to tropical or sidereal mean years and the second peak P2 may associate to non Saronic periodicity (NSP) of "NSP = P2* 29.53059 days) = 411.7863592 days". Metonic lunation (235/P1 ≈ 19 years) and Saronic period (223/P1 ≈ 18 years) have a relation when the moon phase (the new moon and the full moon) occur in a moment of an eclipse. The other periodicity found by Espenak [5] such as 99 lunation or "8 solar mean years" period, Tritos (135 lunation/P1 ≈ 11 solar mean years), Inex (358 lunation/P1 ≈ 29 solar mean years), Exeligmos or Triple Saros (669 lunation/P1 ≈ 54 solar mean solar years + 33 days), or longer period of 4601 lunation/P1 ≈ 372 solar mean years may be found. Non Saronic period for example 251 lunation/P2 ≈ 18 NSP unit equal to 269 anomalistic month.

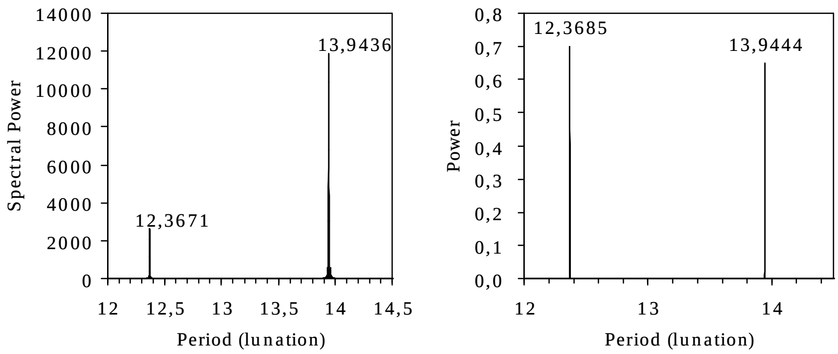

Figure 2. Periodicity of the length of synodic using WCA formula (right) and LSA formula (left)

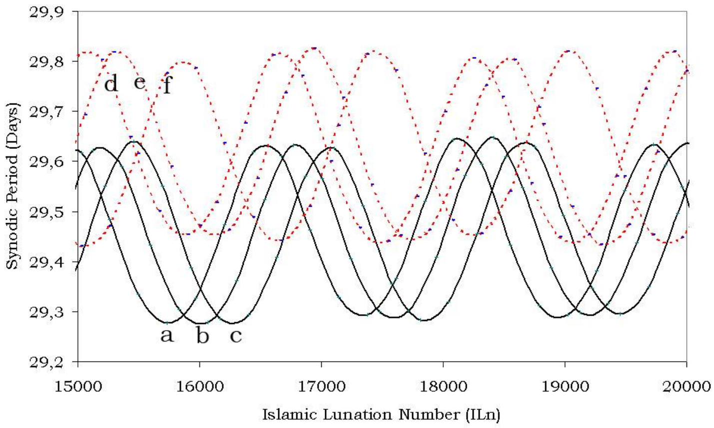

Figure 3. Plot of Synodic Period (days) againts ILn for the calculated data of hilal closed to aphelion group (AG) (solid line), ILVn 223 (a), ILVn 87(b), ILVn 186(c), and closed to perihelion group (PG) (dash-line), ILVn 155 (d), ILVn 19 (e), ILVn 217 (f)

Figure 3 presented plot of Synodic Period (days) againts ILn, the range of the synodic period when the earth closed to aphelion are compared with the synodic period when the earth closed to perihelion. The range of synodic period of AG is between 29,27531 - 29,65460 days, with average 29,44941 days. The range of synodic period of AG is shorter than the range of synodic period of PG, between 29,42718 - 29,82933 days, with average 29,61243 days. It can be explained that the speed of the earth revolution in perihelion is larger than in aphelion so the moon needs longer time to finish one synodic period.

The range of synodic period when the moon closed to equinoxes provided in Figure 4, both cases have almost similar synodic period either in March or in September. The range of periodicity has similarity for both Vernal Equinox Group (VEG) and Autumnal Equinox Group (AEG). The AEG has range of synodic period between 29,33186 - 29,74438 days, with average 29,52301 days and VEG has synodic range between 29,34470 - 29,76568 days, with average 29,53928 days. This condition is due the declination of the Sun almost the same (close to celestial equator) and speed of the earth is almost the same.

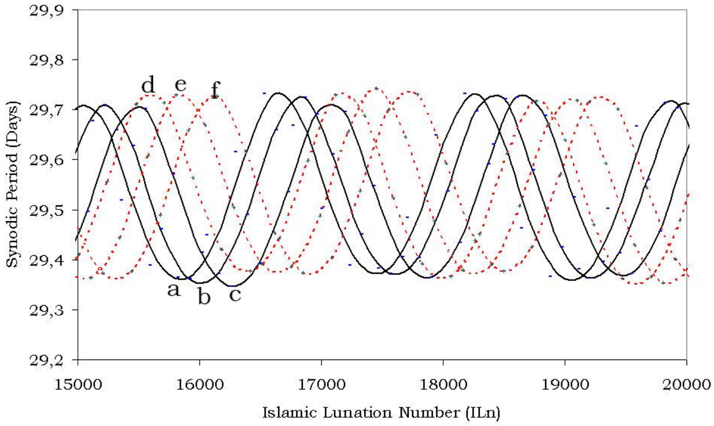

Figure 4. Plot of Synodic Period (days) againts ILn for the calculated data of hilal closed to Vernal Equinox or Spring Equinox or March equinox (solid line) called Vernal Equinox Group (VEG) composed of ILVn 71 (a), ILVn 170 (b) and ILVn 34 (c), and closed to Autumnal Equinox or September equinox (dash-line) called Autumnal Equinox Group (AEG) composed of ILVn 201 (d), ILVn 65 (e) and ILVn 164(f)

The changing of synodic period has the same range between minimum and maximum value of synodic period for both cases. It is interesting, both cases are in same condition, the sun is exactly in celestial equator, the speed is almost the same or it doesn't influence to synodic period such as when the earth closed in perihelion or aphelion passage.

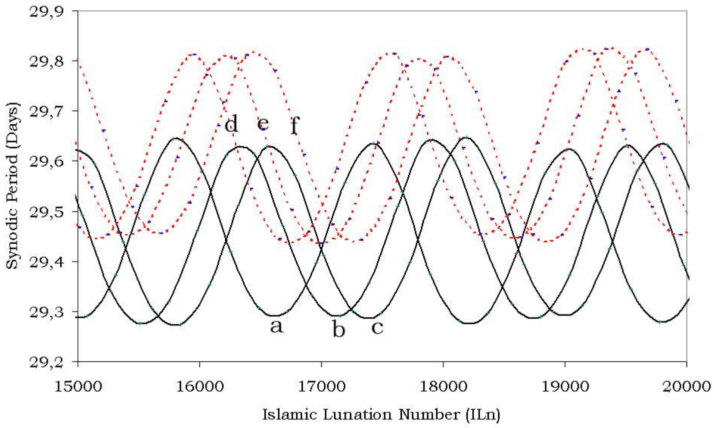

Figure 5. Plot of Synodic Period (days) againts ILn for the calculated data of hilal closed to Summer Solstice or June solstice (solid line) called Summer solstice Group (SSG) composed of ILVn 37 (a), ILVn 235 (b) and ILVn 99 (c), and closed to Winter Solstice or December solstice

(dash-line) called Winter solstice Group (WSG) composed of ILVn 204 (d), ILVn 68 (e) and ILVn 167 (f).

The range of synodic period when the moon closed to solstices, SSG and WSG, provided in Figure 5. The range of periodicity when the moon closed to summer solstice or SSG is between 29,27182 − 29,64773 days, with average is 29,44783 days, and winter Solstice between or WSG is 29,43294 − 29,83271 days, with average is 29,61992 days. The declination of the Sun for both solstices, summer and winter solstice, is difference. The Sun is located in extreme north (in June) or declination around +23.5 degree and extreme south (in December) or declination of the Sun around −23.5 degree. The longest synodic period will be happened when the Sun is located in the most southern sky; it will be happened when the earth approaches to perihelion passage. It will be shorter when the Sun is located in the most northern sky, it will be happened when the earth approaches to aphelion passage.

4 Conclusions

The Saronic (223 lunation period) and the Metonic (235 lunation period) cycle are important cycle to study further regularity on earth-moon-sun (EMS) phenomena. The regularity of in internal Metonic group or ILn belongs to ILVn has been shown for the position of the Sun closed to equinox and soltices, as well as when the earth closed to perihelion, and aphelion. The Sun position as a reflection on the speed of the earth moves around the Sun will affect the length of synodic period as well as the range of synodic period. The Moon orbital complexity is reflected to the pattern of regularity in the plot of synodic period againts ILn for each ILVn. The gradual change of synodic period for all group of ILVn in Figure 1 is clearly composed of regularity of each ILVn.

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