Finite element analysis of composite pressure vessels with a load sharing metallic liner

Mohammad Z. Kabir
Department of Civil Engineering, AmirKabir University of Technology, Tehran, Iran

Abstract

A numerical analysis of filament-reinforced internally pressurized cylindrical vessels with over-wrapped metallic liner is presented. The method uses the load-bearing liner approach and leak-before-rupture as design criteria. The structure is modeled as an elastic, ideally plastic liner-reinforced with a quasi-isotropic elastic composite. Based on a balanced stress condition in the pressure vessel, the head shape is obtained by a numerical solution of an elliptic integral. The winding process creates a variable thickness in dome area and results in considerable changes in the on-axis stress distribution incorporated in this study. A 3-D, 2-node interface element is also used to model contact at discrete points between the metallic liner and its surrounded composite shell. Numerical results are reported for the effects of different head shapes and the superiority of optimum geodesic head shapes in reducing the maximum stresses is also investigated.

Incorporating the metallic liner in the analysis produces marked changes in on-axis stresses and resultant displacements.

© 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Finite element analysis; Composite pressure vessels; Load sharing metallic liner

1. Introduction

Modern composites, using continuous fibres in a resin matrix, are important candidate materials in the engineering of energy-efficient structures. In many applications, fibre/matrix materials are lighter, stronger and more cost effective when compared with traditional materials like metals. Many high-strength composite products are fabricated using the filament winding process. In this process, bands of resin-impregnated fibre are wound over a cylindrical mandrel using a computer controlled fibre placement machine. Fibre application to the metallic mandrel is usually accomplished by a transverse feed head over a rotating surface, capable of achieving any desired winding orientation. Filament-wound tubular structures, more specifically pressure vessels, offer significant weight saving over conventional all metallic ones for containment of high pressure gases and liquids.

A rational engineering procedure has evolved for minimum weight design of filament-reinforced metal-lined vessels which couples strength and strain compatibility analyses with experimental data. In aerospace applications, composite tanks with load-sharing liners provide significant weight saving on the order of 40% or more over the highest performance of homogeneous metal vessels. Based on successful development results and the need to save weight, composite pressure vessels have found many applications in space, missile and aircraft systems.

A number of authors have addressed various structural problems confronting filament-wound, metal-lined cylinders under internal pressure. Most of these analyses have concentrated on relatively thin-walled structures with low-to-moderate storage capability. Solutions for multilayer pressure vessels, based on elastic limits in connection with optimum design, have also been discussed in several investigations. Early work employed netting analysis to arrive at efficient filament wrap angles and end-closure shapes [6,4]. Later papers have dealt with the liner plastic flow required for the more compliant composite to achieve full strength and its effects on cyclic burst strength, [1,3]. Sabbaghian and Nandan [5] have used the maximum shear theory to determine the optimum relations between internal pressure, radial and tangential stresses. The numerous studies cited so far focused much on designing the composite pipes and neglected the dome-ended effect on the winding path, stress concentration at the junction of cylinder and cap and analyzing the entire hybrid structures which is the main issue of the present study. One of the principal factors influencing the integrity of
filament wound structures is void ratio, i.e. the number of air pockets trapped within the matrix on manufacture. Ideally, a filament-wound structures should follow the mandrel surface completely with no voids or twisting in the process. In other words, to achieve the most efficient use of the reinforcement material, the fibres may slip during the winding process. This limits design optimization as the winding trajectories are constrained to follow near geodesic curves, the shortest path between two points on a curved surface, to prevent fibre slippage.

Stressing of the filament-wound composite results in the formation of some matrix cracks in the wall that can let the contained fluid leak out. To prevent this, a liner is required. Three classes of liners used are:

1. Elastomeric, for near ambient temperature applications where some permeability is permissible.
2. Thin-metal bonded to over-wrap, the lightest weight vessel with limited cyclic life which is used in this study.
3. Load sharing metal with cyclic life performance are intermediate in weight saving performance between thin-metal/bonded liner composite tanks and homogeneous metal tanks.

The main attention of the present work is to measure the on-axis stresses and reference corresponding displacements created by internal pressure in a hybrid metal-composite pressure vessel with geodesic dome shape and variable thickness and to compare the results from this study with those which could be obtained with traditional solutions.

2. Analytical approach

2.1. Geodesic path

The term “geodesic isotenoid pressure vessel” is applied to pressure vessel consisting entirely of filaments that are loaded to identical stress levels. The theory of such a pressure vessel under internal pressure has been discussed by Kitzmiller et al. [2] and was applied by them appropriately to head shapes without openings. A generalization of their results requires the filaments lie along geodesic lines. In the region of the cap, the filament path is normally adapted to helical winding. Using shell theory [7] for balanced stresses in the dome, the fibre path needs to satisfy the following condition, Fig. 1

$$\frac{N_\phi}{R_1} + \frac{N_\theta}{R_2} = P$$  \hspace{1cm} (1)

The meridian and circumferential radii, $R_1$ and $R_2$, respectively, are defined [6] as follows:

$$R_1 = -\frac{[1 + y^2]}{y''}, \hspace{1cm} R_2 = -x \frac{[1 + y^2]^2}{y'}$$ \hspace{1cm} (2)

Fig. 1. Geometrical illustration of filament wound cylindrical pressure vessels. (a) Free body diagram between membrane forces and internal pressure (b) Helical winding in cylindrical pressure vessels (c) Geometry of a filament-wound dome.
where $y'$ and $y''$ are the first and second derivatives of $y$ with respect to $x$, respectively. $x$ and $y$ are the coordinates of each point on the contour. $N_\phi$ and $N_\theta$ are the meridional and circumferential forces, respectively, and are defined using the previous transforming relations

$$N_\phi = \sigma_\phi r \cos^2 \alpha, \quad N_\theta = \sigma_\theta r \sin^2 \alpha,$$

where $\alpha$ is the winding angle, (Fig. 1), and $\sigma_\phi$ the ultimate tensile strength of the composite in the fibre direction. The relation between meridian and circumferential radii are simplified as

$$\frac{R_2}{R_1} = 2 - \tan^2 \alpha.$$  

Eq. (4) is valid for $\tan^2 \alpha < 2$ and can be transformed into

$$\frac{xy''}{y'(1 + y'^2)^{1/2}} = 2 - \tan^2 \alpha.$$  

The condition of filaments lying along geodesic lines holds true, Fig. 1, when

$$X \sin \alpha = X_0 = \text{Const.},$$  

where $X_0$ is the boss radius. If the filaments are to be wound continuously for an opening $x_0$, then the constant in Eq. (6) can be evaluated, because the filament must be tangent at the opening. Consequently

$$\sin \alpha = \frac{X_0}{x}.$$  

Substituting Eq. (7) into Eq. (5) gives the following differential equation

$$\frac{xy''}{y'(1 + y'^2)^{1/2}} = \frac{2x^2 - 3X_0^2}{x^2 - X_0^2}.$$  

The solution of the above equation becomes the following elliptic integral of the third kind which can be solved by a computer [8]

$$y = -\int \frac{x^3 \, dt}{[(1 - x^2)(x^2 - a_1)(x^2 - a_2)]^{1/2}} + C,$$

where

$$a_1 = \frac{1}{2} \left[ \frac{1 + 4X_0^2}{1 - X_0^2} \right]^{1/2} - 1,$$

$$a_2 = \frac{1}{2} \left[ \frac{1 + 4X_0^2}{1 - X_0^2} \right]^{1/2} + 1.$$  

The constant of integration is evaluated from the fact that $y = 0$ when $x = 1$.

2.2. Load-sharing thin metallic liner

For vessels with metallic liners, the filament stress–strain curve is linearly elastic to the proof strain and even beyond to bursting. However, on the pressurization cycle, the metal stress–strain curve shows yield and plastic flow, Fig. 2, as the liner is forced into compression by filaments trying to return elastically to their original size. Thus, at zero pressure after proof loading, the metal is in compression and the filaments are in tension. Thus, the metal operates elastically from compression to tension while the filaments operate in a tension–tension mode. The primary objectives in designing a fibre reinforced metal pressure vessel are, therefore, to obtain maximum operating performance at a minimum weight and to provide safe-life design features. Fig. 2 shows the loading and unloading cycles. Design development is related to

(a) Equilibrium and strain compatibility of two types of materials are defined as follows, respectively.

$$\sigma_\phi + \sigma_c = PR,$$

$$\sigma_c = \left( \frac{E_c}{E_1} \right) \left( \frac{1 - \nu_l}{1 - \nu_c} \right) \sigma_l,$$

where $\sigma_l$ and $\sigma_c$ are average hoop stresses, $\nu_l$ and $\nu_c$ are poisson ratios in liner and composite, respectively, $t_l$ and $t_c$ are the thicknesses of liner and composite, respectively. $P$ is the internal pressure and $R$ the internal radius of the metal liner, $E_l$ liner elastic modulus and $E_c$ composite equivalent elastic modulus in reference axes, axial and hoop directions, and is introduced in terms of on-axis material principal constants in the following forms, respectively.

![Fig. 2. Metal-composite relationship in load sharing filament-reinforced metal composite cylinders.](image-url)
(b) Metal shell compressive strength is adequate so that adhesive bonding is not required to prevent metal shell buckling. The minimum liner thickness against buckling may be calculated from the following equation

$$t_l = \frac{3\sigma_l(1 - \nu^2)^{1/2}}{E_l} - R. \quad (13)$$

(c) Fatigue control is obtained under following compatibility equation

$$\frac{n}{N} + \frac{\epsilon_b}{\epsilon} \leq 1. \quad (14)$$

In this equation, $n$ is the loading cycles, $N$ is the number of fatigue cycles at the maximum operating strain range that would produce fatigue failure, $\epsilon$ the maximum strain capability of the virgin material and $\epsilon_b$ the strain at required burst pressure.

2.3. Finite element modelling

The finite element software, Numerical Integrated System Analysis (NISA-II), is employed in this study. The 3D laminated composite general shell element, NKTP = 32, is chosen for this purpose. NISA-II translates the loads (internal pressure), boundary conditions and material specification from the finite element pre- and post-processor code for an accurate stress analysis of the cylinder. The 3D shell element includes the deformations due to membrane, bending and membrane-bending coupling and transverse shear effects and is

Fig. 3. Finite element modeling assembly of metal/gap/composite pressure vessel. (a) 3D gap/friction element (NKTP = 50) (b) Finite element mesh for metal-composite pressure vessels.
suited for modelling moderately thick to thin laminated composite shells. The element consists of a number of layers of perfectly bonded, orthotropic materials. The 3D general isotropic shell element, \( \text{NKTP} = 20 \), with nonlinear capability and incorporating Von Mises elastic–perfect plastic nonlinear behaviour for the metal liner, is also used. The 3D Gap/Friction element, \( \text{NKTP} = 50 \), is a 2-node interface element which may be used in 3D problems to model contact at discrete points between two bodies, is inserted between metal liner and composite shell. The Gap element has the translational degrees of freedom \( (U_x, U_y, U_z) \) at each node. The element is nonlinear and can resist normal compressive force and tangential shear forces which are represented by coupled nonlinear springs, one in normal direction and two in orthogonal tangential directions to interface. The axial stiffness of the gap, \( K_n \), is taken 3 orders of magnitude higher than the stiffness of the adjacent elements, composite or steel; the tangential, \( K_r \) is taken equal to \( K_n \). The element configuration and finite element mesh for the half-cylinder is shown in Fig. 3(a) and (b).

### 2.4. Geometrical consideration on head shape region

A winding pattern includes many wraps with different wrap angles. The desirable winding pattern, as explained earlier to satisfy isotensoid condition, is calculated from Eq. (6). The determination of the overall geometry and elastic constant of a composite involves the calculation of the local wrap angle of each wrap and its corresponding cross sectional thickness. By assuming the wrap angle at the juncture of cylinder and head shape, \( \alpha_0 \), and using Eq. (6), the local wrap angle corresponding to any point on the geodesic profile, \( (x, y) \), can be determined as

\[
\sin \alpha_n = \frac{R}{x} \sin \alpha_0.
\]

where \( R \) is the cylindrical internal radius and \( x \) a radial distance from the longitudinal axis at each level to maximum value \( R \). The curve of the head shape has a point of inflection at \( x = 1.22X_0 \) and Eq. (9) is not applicable for smaller \( X_0 \). Consequently, for region \( X_0 \leq x \leq 1.22X_0 \), the vicinity of the opening, additional reinforcement is required in the form of an insert to distribute the meridional load. In this study, for sake of simplicity, the head shape curve for this region is obtained as a tangent to the geodesic curve at \( x = 1.22X_0 \). A cross section of the over-wrap along the reference meridian is shown in Fig. 4. The local thickness \( (t_n) \) at \( x = 1.22X_0 \) is given by

\[
t_n = \frac{R \cos \alpha_0}{x \cos \alpha_n} t_0,
\]

where \( t_0 \) is the total thickness of the helical layers and can be determined from basic thin-walled pressure vessels relations in terms of internal pressure, \( P \), ultimate strength of the composite in fibre direction, \( \sigma_u \), and winding angle, \( \alpha_n \), as

\[
t_0 = \frac{PR}{2\sigma_u \cos^2 \alpha_0}.
\]

### 3. Numerical evaluation

The hybrid pressure vessel comprised of a stainless steel liner reinforced by a Kevlar 49/Epoxy composite has a 150 mm inside diameter with a 50 mm diameter at the polar boss, the total length is 742 mm and the cylindrical portion is 642 mm. The maximum operating pressure is 22 Mpa and the proof pressure factor is 1.5. This study ignores the cycles for fatigue life. The elastic constants of the composite material are: \( E_{11} = 90 \) GPa, \( E_{22} = 4.115 \) GPa, \( v_{12} = 0.29 \), \( G_{12} = 1.8 \) GPa, and

<table>
<thead>
<tr>
<th>( y ) (mm)</th>
<th>( x ) (mm)</th>
<th>Thickness ( (t_n) ) (mm)</th>
<th>Winding angle ( \alpha_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75</td>
<td>2.373</td>
<td>19.5</td>
</tr>
<tr>
<td>6</td>
<td>74.54</td>
<td>2.427</td>
<td>19.62</td>
</tr>
<tr>
<td>11.87</td>
<td>73.19</td>
<td>2.595</td>
<td>20</td>
</tr>
<tr>
<td>17.47</td>
<td>71</td>
<td>2.91</td>
<td>20.6</td>
</tr>
<tr>
<td>22.71</td>
<td>68.03</td>
<td>3.443</td>
<td>21.6</td>
</tr>
<tr>
<td>27.5</td>
<td>64.38</td>
<td>4.381</td>
<td>22.9</td>
</tr>
<tr>
<td>31.78</td>
<td>60.15</td>
<td>6.345</td>
<td>24.6</td>
</tr>
<tr>
<td>35.53</td>
<td>55.44</td>
<td>14.955</td>
<td>26.8</td>
</tr>
<tr>
<td>38.74</td>
<td>50.34</td>
<td>11.415</td>
<td>29.8</td>
</tr>
<tr>
<td>41.44</td>
<td>44.96</td>
<td>10.958</td>
<td>33.8</td>
</tr>
<tr>
<td>43.67</td>
<td>39.37</td>
<td>10.467</td>
<td>39.5</td>
</tr>
<tr>
<td>45.41</td>
<td>32</td>
<td>9.937</td>
<td>51.5</td>
</tr>
<tr>
<td>54</td>
<td>25</td>
<td>9.359</td>
<td>90</td>
</tr>
</tbody>
</table>
The elastic constants of the steel liner are: \( E = 4.115 \ G Pa, \ \nu = 0.3, \ \sigma_y = 345 \ MPa \). Using Eq. (7), the winding orientation is obtained \((\pm 19.5^\circ)\) for helical layers in the cylindrical portion. The preliminary calculation of the required helical and circular layers are based on netting analysis and can be found in following equations:

The minimum number of helical layers is:

\[
N_h = \frac{PR}{2t \sigma_0 \cos^2 \alpha}
\]

and the minimum number of circular layers in the cylindrical portion is

\[
N_c = \frac{PR}{2t \sigma_0} (2 - \tan^2 \alpha),
\]

where \( t \) is the fibre band thickness. In this analysis, 6 helical layers and 9 circular layers, in order to satisfy longitudinal and hoop stresses, are calculated. Therefore, the fibre band thickness is 90.5 MPa.

Fig. 5. Comparison of different head shapes for principal on-axis stress distribution.

Fig. 6. Resultant displacement of internally pressurized composite vessel, \( P = 220 \) bars (a) Dome with variable thickness (b) Dome with constant thickness.
Therefore, the cylindrical portion is made up of a total of 15 layers and the cap comprises 6 helical layers. In the head area, the geometrical properties change as the helical angle varies from the tangent plane, 19.5°, to the boss neck, 90°, Eq. (15). The various helical angles and respective material properties, e.g. layer thickness, for all elements in dome section were obtained following the geodesic solution and tabulated in Table 1. The longitudinal stress, $S_{xx}$, representing material principal (on-axis) stress in fibre direction for layer number one (helical winding 19.5°) is depicted in Fig. 5 for three different closed-end shapes, hemispherical, shell cap which is called a dome in the literature and the geodesic head with constant and variable thickness, respectively. It can be observed that at the junction of cylinder and cap, as a result of sudden change in geometry, stress concentrations occurred for the cases of spherical and dome heads. In these cases, early burst failure may take place in this area. Fig. 5 clearly reveals the efficiency of the optimum path by shifting the stress concentration from the critical zone toward the polar opening boss for constant thickness. For variable thickness with the geodesic path, however, it is seen that the maximum stress at the junction between cylinder and cap, from 1430 MPa in the cylinder with dome head, drops to 570 MPa for the cylinder with geodesic head. It can be observed that, for all three head shapes, the stresses in the cylindrical portion are almost the same but, in the neighbourhood of the cylinder/cap junction (about 300 mm from the centre of the vessel), the differences between the stress contours is marked. The geodesic path has a lower fluctuation compared with the spherical and dome heads. The novelty of this work is including variable thickness and winding angle, Eqs. (15) and (16), of the head area in the analysis for geodesic path.

Fig. 6(a) and (b) also compare the resultant displacement of the composite vessel, which is internally pressurized to 220 bars, for constant and variable thickness in the head area. It is seen that incorporating the variable thickness in the analysis limits markedly the deformation of the structures. The development of plastic flow in the metallic liner by increasing of internal pressure is depicted in Fig. 7(a). The horizontal axis is represented the distance from the centre of the vessel.
and the vertical axis represents the maximum effective stress created by internal pressure in the metal liner. As can be observed, plasticity grows from the polar boss opening side towards the cylindrical cap junction. After certain amount of internal pressure, e.g. 110 bars, the total metal liner yields into plastic zone. Fig. 7(b) also shows the history of developing of maximum stress for different stages of pressurizing for two different locations on the vessel, one on the head, node number 342, and the other on the cylinder, node number 681. It is seen that the cap area of the metal liner becomes plastic faster than the cylindrical part. This result may indicate the sensitivity of the cap area in cycling loading for low pressure. Some additional results regarding the load sharing liner can be listed as follows:

1. The axial and radial displacements of three models; elastic–perfectly plastic (considered as lower bound plasticity behaviour) metallic over-wrapped liner with interface gap element, fully elastic (upper bound plasticity limit) liner surrounded by composite shell and a composite pressure vessel without liner are compared with each other through Fig. 8(a) and (b). In general, the analysis of finite element models with interface gap element i.e. liner bonded a composite, results in higher strains in the dome area for the metallic liner with elasto-plastic behaviour for axial deformation, Fig. 8(a), and conversely, in radial deformation for the one without liner, Fig. 8(b).

2. Fig. 9 represents the principal on-axis fibre direction stress, $S_{xx}$, for a longitudinal section of vessel under internal pressure, $P = 220$ bars, in helical layer for, again, the above three cases. It is observed that, incorporating the metallic liner in the analysis, the maximum stress drops from 460 to 330 MPa. For a mandrel with a high elastic modulus, which can be assumed as a fully
elastic liner, the sharing with mandrel in carrying the load is remarkable and it reduces the maximum stress, $S_{xx}$, to 250 MPa. The variation of the principal on-axis transverse stress, $S_{yy}$, and in-plane shear stress, $S_{xy}$, versus longitudinal section of vessel for three mentioned cases are depicted in Figs. 10 and 11, and are both less sensitivity to the influence of the elasto-plastic liner in dropping the maximum stress.

Fig. 12 presents those comparisons in a hoop layer, $\alpha = 90^\circ$. As mentioned, the hoop layer exists only on the cylindrical portion; for the head area only helical layers are possible. Taking into account elasto-plastic and fully plastic behaviour as two extreme cases, it is seen that the reduction of maximum on-axis stress, $S_{xx}$, which takes place at the junction of cylinder and cap, would be from 700 MPa for composite shell without liner, to 500 MPa for the assembly of metal-composite with elasto-plastic liner and 300 MPa for the hybrid assembly with fully elastic liner. The obtained results represent an important role for metallic liners in strengthening composite pressure vessels and could be useful for inclusion in design specifications.

4. Conclusion

For filament wound structures, determination of the elastic constants plays a very important role in the structural analysis since many variables and assumptions are involved. The present work takes the form of a feasibility study investigating the practicality and superiority of end-closure shapes in over-wrapped metallic cylindrical high pressure vessels. Based on a balanced stress conditions, the optimized shapes for the metallic mandrel are examined in this work. To establish the optimum contour path an analytical formulation is postulated leading to the solution of an elliptic integral which yields the coordinates of the closure head. The load-bearing liner approach is introduced as a design criterion for metallic over-wrapped hybrid pressure vessels and is used in the analysis. The numerical results obtained from NISA-II finite element software reveal that:

- Incorporating the variable thickness in the analysis limits considerably the deformations of the structures.
- The metallic liner produces a remarkable drop in the principal on-axis stress, $S_{xx}$, in both helical and hoop wound layers.

Although the methods presented in this report are analytically justifiable, their accuracy must be verified by test data.

Acknowledgements

The author would like to acknowledge financial support from Prof. M. Pandey and assistance from Prof. A.N. Sherbourne for the writing of this manuscript during a short visit to the University of Waterloo.

References